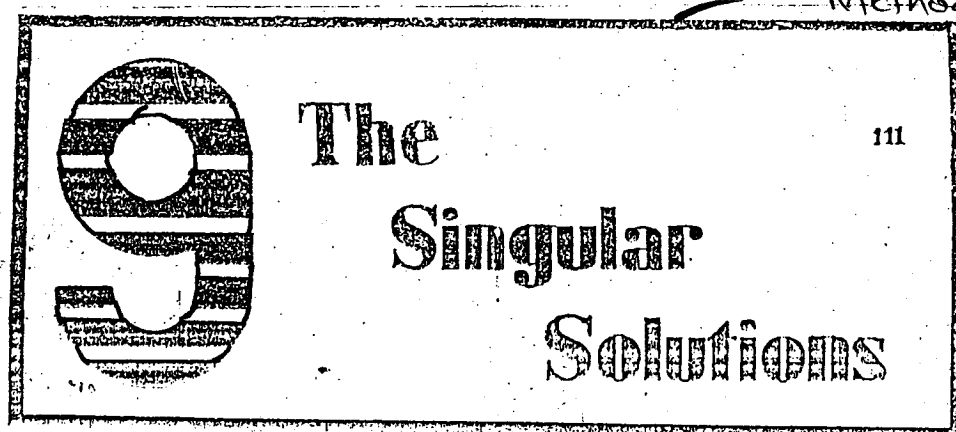




Singular Solution

A diff. eq. $f(x, y, p) = 0$ may possess a solution which does not involve any arbitrary constant and, in general, is not obtained from the general solution by giving any particular value to the arbitrary constants, is called Singular Solution.

P-discriminant

Consider the non-linear diff. eq. of first order, $f(x, y, p) = 0$ ----- ①

Differentiating ① partially w.r.t P , i.e. $\frac{\partial f(x, y, p)}{\partial p} = 0$ ----- ②

If we eliminate P from ① and ② then the eliminant eq. (resulting eq.), is called P-discriminant for eq. ①

Remark

If the eq. ① is quadratic in P i.e. is of the shape $AP^2 + BP + C = 0$ then the P-disc. is given by $B^2 - 4AC = 0$

C-discriminant

✓ Consider the non-linear diff. eq. of first order, $f(x, y, p) = 0$ ----- ①

Let the general solution of ①, be $\phi(x, y, c) = 0$ ----- ②

Different. ② partially w.r.t. c , i.e. $\frac{\partial \phi(x, y, c)}{\partial c} = 0$ ----- ③

If, we eliminate c from ② and ③, we get the eliminated eq. (resulting eq.) called c -disc. for ②

Remark

If eq. ② is quadratic in c , i.e. of the shape $Ac^2 + Bc + C = 0$ then the c -disc. is given by $B^2 - 4AC = 0$

Determination of Sing. Sol.

Suppose, we want to find the singular sol. of diff. eq. $f(x, y, P) = 0$ ——— ①

- ① Find the general solution of ①
- ② Find the c -discriminant
- ③ Find the P -discriminant
- ④ The common part in both the discriminants, that satisfies the diff. eq. ①, is the singular solution of ①

The P disc. Method

We can obtain the singular solution of the diff. eq. $f(x, y, P) = 0$ ——— ① directly from eq. ①, as

Find P -discriminant for ①. The part of this relation that satisfies the diff. eq. ①, is the singular sol. of ①

Example

Solve and find singular sol. of

$$P^2 - xP + y = 0 \text{ ——— ①}$$

Sol:-

$$y = xP - P^2$$

It is Clairaut's eq. so its general sol is $y = cx - c^2$

$$\text{or } c^2 - cx + y = 0 \text{ ——— ②}$$

Singular Sol.

we find the singular sol. of ①, as

Example

Solve and find singular sol. of

$$xP^2 - 2yP + 4x = 0 \text{ ——— ①}$$

Sol:-

$$2yP = xP^2 + 4x$$

$$\Rightarrow 2y = xP + 4xP^{-1}$$

Differentiating w.r.t x

$$2 \frac{dy}{dx} = x \frac{dP}{dx} + P + 4(-xP^{-2} \frac{dP}{dx} + P^{-1})$$

Example

Solve and find singular sol. of
 $(x^2-1)P^2 - 2xyP - x^2 = 0$ ——— ①

Sol:-

$$2xyP = (x^2-1)P^2 - x^2$$

$$\Rightarrow 2xyP = x^2P^2 - P^2 - x^2$$

$$\Rightarrow 2y = xP - x^{-1}P - xP^{-1}$$

Diff. w.r.t x , we get

$$2 \frac{dy}{dx} = x \frac{dP}{dx} + P - (x^{-1} \frac{dP}{dx} - x^{-2}P) - (-xP^{-2} \frac{dP}{dx} + P^{-1})$$

$$\Rightarrow 2P = x \frac{dP}{dx} + P - \frac{1}{x} \frac{dP}{dx} + \frac{P}{x^2} + \frac{x}{P^2} \frac{dP}{dx} - \frac{1}{P}$$

$$\Rightarrow P + \frac{1}{P} - \frac{P}{x^2} = (x - \frac{1}{x} + \frac{x}{P^2}) \frac{dP}{dx}$$

$$\Rightarrow P(1 + \frac{1}{P^2} - \frac{1}{x^2}) = x(1 - \frac{1}{x^2} + \frac{1}{P^2}) \frac{dP}{dx} = 0$$

$$\Rightarrow (1 + \frac{1}{P^2} - \frac{1}{x^2})(P - x \frac{dP}{dx}) = 0$$

$$\Rightarrow 1 + \frac{1}{P^2} - \frac{1}{x^2} = 0 \text{ or } P - x \frac{dP}{dx} = 0$$

Consider,

$$P - x \frac{dP}{dx} = 0$$

$$\Rightarrow x \left| \frac{dP}{dx} \right| = |P|$$

$$\Rightarrow \int \frac{dP}{P} = \int \frac{dx}{x}$$

$$\Rightarrow \ln P = \ln x + \ln c$$

$$\Rightarrow P = cx$$

putting in eq. ①, we get

$$(x^2-1)c^2x^2 - 2cx^2y - x^2 = 0 \text{ ——— ②}$$

(req. general sol. of ①)

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$$(-2y)^2 - 4x \cdot 4x = 0$$

$$\Rightarrow y^2 = 4x$$

Since c-disc. and P-disc. are same

$\therefore y^2 = 4x$ is the singular sol. of ①

Example

By finding the P-disc. find the singular sol. of

$$x^3P^2 + x^2yP + \alpha^3 = 0 \text{ ——— ①}$$

Sol:-

P-discriminant

P-disc. of ①, is given as

$$B^2 - 4AC = 0$$

$$\Rightarrow x^4y^2 - 4\alpha^3x^3 = 0$$

$$\Rightarrow x^3(xy^2 - 4\alpha^3) = 0$$

$$\Rightarrow x^3 = 0 \text{ or } xy^2 - 4\alpha^3 = 0$$

$$\Rightarrow x = 0 \text{ or } xy^2 - 4\alpha^3 = 0$$

Take $x = 0$

Since $x = 0 \therefore \frac{dx}{dy} = 0$

$$\text{or } \frac{1}{P} = 0$$

we, first write ①, as

$$x^3 + x^2y \cdot \frac{1}{P} + \alpha^3 \left(\frac{1}{P}\right)^2 = 0 \text{ ——— ②}$$

L.H.S. of eq. ②

$$= x^3 + x^2y \cdot \frac{1}{P} + \alpha^3 \left(\frac{1}{P}\right)^2$$

$$= 0 + 0 + 0 + 0$$

$$= 0 = \text{R.H.S. of eq. 2}$$

$\therefore x = 0$ is a singular sol. of ①

C-discriminant

C-disc. for ②, is given by

$$\begin{aligned} B^2 - 4AC &= 0 \\ \Rightarrow (-x)^2 - 4y &= 0 \\ \Rightarrow x^2 &= 4y \end{aligned}$$

P-discriminant

P-disc. for ②, is given by

$$\begin{aligned} B^2 - 4AC &= 0 \\ \Rightarrow (-x)^2 - 4y &= 0 \\ \Rightarrow x^2 &= 4y \end{aligned}$$

Since C-disc. and P-disc. are same

$\therefore x^2 = 4y$ is singular sol. of ①

Example

Solve $x^2 P^2 + yP(2x+y) + y^2 = 0$ — ①
by making the substitutions

$$y = u, \quad xy = v$$

and find the singular sol.
Sol:-

$$\text{Here } y = u, \quad x = \frac{v}{u}$$

$$\therefore dy = du, \quad dx = \frac{u dv - v du}{u^2}$$

$$\text{Now } \frac{dy}{dx} = \frac{u^2 du}{u dv - v du}$$

$$\Rightarrow P = \frac{u^2 du}{u dv - v du}$$

Hence eq. ① becomes, as

$$x^2 \left(\frac{u^2 du}{u dv - v du} \right)^2 + y \left(\frac{u^2 du}{u dv - v du} \right) + y^2 = 0$$

$$\Rightarrow 2P = x \frac{dP}{dx} + P - \frac{4x}{P^2} \frac{dP}{dx} + \frac{1}{P}$$

$$\Rightarrow P - \frac{4}{P} = x \left(1 - \frac{4}{P^2} \right) \frac{dP}{dx}$$

$$\Rightarrow P \left(1 - \frac{4}{P^2} \right) - x \left(1 - \frac{4}{P^2} \right) \frac{dP}{dx} = 0$$

$$\Rightarrow \left(1 - \frac{4}{P^2} \right) \left(P - x \frac{dP}{dx} \right) = 0$$

$$\Rightarrow 1 - \frac{4}{P^2} = 0 \quad \text{or} \quad P - x \frac{dP}{dx} = 0$$

Consider,

$$P - x \frac{dP}{dx} = 0$$

$$\Rightarrow x \frac{dP}{dx} = P$$

$$\Rightarrow \int \frac{dP}{P} = \int \frac{dx}{x}$$

$$\Rightarrow \ln P = \ln x + \ln c$$

$$\Rightarrow P = cx$$

putting in ①, we get

$$x \cdot c^2 x^2 - 2y \cdot cx + 4x = 0$$

$$\Rightarrow c^2 x^2 - 2yc + 4 = 0 \quad \text{--- (2)} \\ \text{(general sol. of ①)}$$

Singular Sol:-

We find the singular sol of ① as

C-discriminant

C-disc. of ② is given as

$$B^2 - 4AC = 0$$

$$\Rightarrow (2y)^2 - 4x^2 \cdot 4 = 0$$

$$\Rightarrow y^2 = 4x^2$$

P-discriminant

P-disc. of ①, is given by

$$B^2 - 4AC = 0$$

Singular Sol:

we find singular sol. of ① as,

C-discriminant

C-disc. of ②, is given by

$$B^2 - 4AC = 0$$

$$\Rightarrow (-2xy)^2 - 4x^2(x^2-1)(-x^2) = 0$$

$$\Rightarrow y^2 + x^2 - 1 = 0$$

$$\Rightarrow x^2 + y^2 = 1$$

P-discriminant

P-disc. of ① is given by

$$B^2 - 4AC = 0$$

$$\Rightarrow (-2xy)^2 - 4(x^2-1)(-x^2) = 0$$

$$\Rightarrow y^2 + x^2 - 1 = 0$$

$$\Rightarrow x^2 + y^2 = 1$$

Since C-disc. and P-disc.

are same

$\therefore x^2 + y^2 = 1$ is a singular sol. of ①

Take $y^2x - 4a^3 = 0$

$$\Rightarrow x = 4a^3y^{-2}$$

Diff. w.r.t. y, we get

$$\frac{dx}{dy} = -8a^3y^{-3}$$

$$\Rightarrow \frac{1}{p} = -\frac{8a^3}{y^3}$$

Now

L.H.S. of eq. ②

$$= x^3 + x^2y \cdot \frac{1}{p} + a^3 \left(\frac{1}{p}\right)^2$$

$$= \left(\frac{4a^3}{y^2}\right)^3 + \left(\frac{4a^3}{y^2}\right)^2 \cdot y \cdot \left(-\frac{8a^3}{y^3}\right) + a^3 \left(-\frac{8a^3}{y^3}\right)^2$$

$$= \frac{64a^9}{y^6} - \frac{128a^9}{y^6} + \frac{64a^9}{y^6}$$

$$= \frac{64a^9 - 128a^9 + 64a^9}{y^6}$$

$$= 0$$

= R.H.S. of eq. ②

i.e. eq. ② is satisfied

by $y^2x - 4a^3 = 0$

$\therefore y^2x - 4a^3 = 0$ is a singular sol. of ①

EXERCISE 9.9

1 Solve and find singular sol. of $y = Px + P^n$ ——— ①

Sol:- It is Clairaut's eq.

so its general sol. is

$$y = cx + c^n \text{ ——— ②}$$

Singular sol.

we find singular sol. of ①, as

C-discriminant

Diff. ② partially w.r.t c, we get

$$0 = x + n c^{n-1}$$

$$\Rightarrow c = \left(-\frac{x}{n}\right)^{\frac{1}{n-1}}$$

putting value of c in ②, we get

$$y = \left(-\frac{x}{n}\right)^{\frac{1}{n-1}} x + \left(-\frac{x}{n}\right)^{\frac{n}{n-1}}$$

P-discriminant

Diff. ① partially w.r.t. P, we get

$$0 = x + n P^{n-1}$$

$$\Rightarrow P = \left(-\frac{x}{n}\right)^{\frac{1}{n-1}}$$

putting value of P in ①, we get

$$y = \left(-\frac{x}{n}\right)^{\frac{1}{n-1}} x + \left(-\frac{x}{n}\right)^{\frac{n}{n-1}}$$

Since C-disc. and P-disc are same

$$y = \left(-\frac{x}{n}\right)^{\frac{1}{n-1}} x + \left(-\frac{x}{n}\right)^{\frac{n}{n-1}}$$

is a singular sol. of ①

2 Solve and find sing. sol. of $P^2(x^2 - a^2) - 2Pxy + y^2 - b^2 = 0$ ——— ①

Sol:-

$$P^2 x^2 - a^2 P^2 - 2Pxy + y^2 - b^2 = 0$$

It is not solvable for P, X, Y

So, we write it as

$$P^2 x^2 - 2Pxy + y^2 - a^2 P^2 - b^2 = 0$$

$$\Rightarrow P^2 x^2 - 2Pxy + y^2 = a^2 P^2 + b^2$$

$$\Rightarrow (Px - y)^2 = a^2 P^2 + b^2$$

$$\Rightarrow Px - y = \pm \sqrt{a^2 P^2 + b^2}$$

$$\Rightarrow y = xP \pm \sqrt{a^2 P^2 + b^2}$$

It is Clairaut's eq.

so its general sol. is,

$$y = cx \pm \sqrt{a^2 c^2 + b^2}$$

$$\Rightarrow y - cx = \pm \sqrt{a^2 c^2 + b^2}$$

$$\Rightarrow (y - cx)^2 = a^2 c^2 + b^2$$

$$\Rightarrow y^2 + c^2 x^2 - 2cxy = a^2 c^2 + b^2$$

$$\Rightarrow c^2 x^2 - a^2 c^2 - 2cxy + y^2 - b^2 = 0$$

$$\Rightarrow (x^2 - a^2)c^2 - 2xy c + (y^2 - b^2) = 0 \text{ ——— ②}$$

Singular sol.

We now find the S. sol. of ① as,

C-discriminant.

c-disc. of eq. ② is given by

$$B^2 - 4AC = 0$$

$$\Rightarrow (-2xy)^2 - 4(x^2 - a^2)(y^2 - b^2) = 0$$

$$\Rightarrow 4x^2 y^2 - 4(x^2 y^2 - b^2 x^2 - a^2 y^2 + a^2 b^2) = 0$$

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$$xP^2 + (x-y)P + 1-y = 0 \quad \text{--- ①}$$

Sol:-

$$xP^2 + xP - yP + 1 - y = 0$$

It is not solvable for x, y, P

So, we write it as,

$$xP(P+1) - y(P+1) + 1 = 0$$

$$\Rightarrow xP - y + \frac{1}{P+1} = 0$$

$$\Rightarrow y = xP + \frac{1}{P+1}$$

It is Clairaut's eq. so, its

$$\text{general sol. is } y = cx + \frac{1}{c+1}$$

$$\Rightarrow y(c+1) = c(c+1)x + 1$$

$$\Rightarrow cy + y = c^2x + cx + 1$$

$$\Rightarrow c^2x + cx - cy + 1 - y = 0$$

$$\Rightarrow c^2x + c(x-y) + (1-y) = 0 \quad \text{--- ②}$$

Singular Sol:-

We find singular sol of ①, as

C-discriminant

c-disc. of ② is, given by

$$B^2 - 4AC = 0$$

$$\Rightarrow (x-y)^2 - 4x(1-y) = 0$$

$$\Rightarrow x^2 + y^2 - 2xy - 4x + 4xy = 0$$

$$\Rightarrow x^2 + y^2 + 2xy - 4x = 0$$

$$\Rightarrow (x+y)^2 - 4x = 0$$

P-discriminant

P-disc. of ①, is given by

$$B^2 - 4AC = 0$$

$$\Rightarrow (x-y)^2 - 4x(1-y) = 0$$

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$$\Rightarrow 4x^2y^2 - 4x^2y^2 + 4b^2x^2 + 4a^2y^2 - 4ab^2 = 0$$

$$\Rightarrow b^2x^2 + a^2y^2 = a^2b^2$$

P-discriminant

P-disc. of eq. ① is given by

$$B^2 - 4AC = 0$$

$$\Rightarrow (-2xy)^2 - 4(x^2 - a^2)(y^2 - b^2) = 0$$

$$\Rightarrow 4x^2y^2 - 4(x^2y^2 - b^2x^2 - a^2y^2 + a^2b^2) = 0$$

$$\Rightarrow x^2y^2 - x^2y^2 + b^2x^2 + a^2y^2 - a^2b^2 = 0$$

$$\Rightarrow b^2x^2 + a^2y^2 = a^2b^2$$

Since c-disc. and P-disc. are same

$$\therefore b^2x^2 + a^2y^2 = a^2b^2 \text{ is the}$$

Singular sol. of eq. ①

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$$4P^2 = 9x \quad \text{--- ①}$$

Sol:-

$$x = \frac{4}{9}P^2$$

Diff. it w.r.t y , we get

$$\frac{dx}{dy} = \frac{4}{9} \cdot 2P \frac{dP}{dy}$$

$$\Rightarrow \frac{1}{P} = \frac{8}{9}P \frac{dP}{dy}$$

$$\Rightarrow 9dy = 8P^2dP$$

$$\Rightarrow \int 8P^2dP = \int 9dy$$

$$\Rightarrow \frac{8}{3}P^3 = 9y + C_1$$

$$\Rightarrow \frac{8}{27}P^3 = y + C_1/9$$

$$\Rightarrow \left(\frac{2}{3}P\right)^3 = y + C$$

$$\Rightarrow \frac{2}{3}P = (y + C)^{1/3}$$

$$\Rightarrow P = \frac{3}{2}(y + C)^{1/3}$$

$$\Rightarrow x^2 + y^2 - 2xy - 4x + 4xy = 0$$

$$\Rightarrow x^2 + y^2 + 2xy - 4x = 0$$

$$\Rightarrow (x+y)^2 - 4x = 0$$

Since C-disc. and P-disc.

are same. $\therefore$ singular sol.

of ① is $(x+y)^2 - 4x = 0$

5 $4xP^2 = (3x-1)^2$

Sol:-

$$4xP^2 = (3x-1)^2 \text{ --- ①}$$

$$P^2 = \frac{(3x-1)^2}{4x}$$

$$\sqrt{P^2} = \frac{\sqrt{(3x-1)^2}}{2\sqrt{x}}$$

$$P = \pm \frac{(3x-1)}{2\sqrt{x}}$$

$$\frac{dy}{dx} = \pm \left[\frac{3x}{2\sqrt{x}} - \frac{1}{2\sqrt{x}} \right] \because \frac{1}{P} = \frac{dx}{dy}$$

$$dy = \pm \left[\frac{3}{2} x^{1/2} - \frac{1}{2} x^{-1/2} \right] dx$$

$$\int dy = \pm \int \left[\frac{3}{2} x^{1/2} - \frac{1}{2} x^{-1/2} \right] dx$$

$$\Rightarrow y = \pm \left[\int \frac{3}{2} x^{1/2} dx - \int \frac{1}{2} x^{-1/2} dx \right]$$

$$\Rightarrow y = \pm \left[x^{3/2} - x^{1/2} + c \right]$$

$$\Rightarrow y = \pm \left[x^{3/2} - x^{1/2} \right] - c$$

$$y+c = \pm \left[x^{3/2} - x^{1/2} \right]$$

$$(y+c)^2 = \left[x^{3/2} - x^{1/2} \right]^2$$

$$(y+c)^2 = x^3 + x - 2x^2$$

$$(y+c)^2 = x(x^2 + 1 - 2x)$$

$$y^2 + c^2 + 2cy = x(x-1)^2$$

$$c^2 + 2cy + y^2 - x(x-1)^2 = 0$$

$$c^2 + 2yc + [y^2 - x(x-1)^2] = 0 \text{ --- ②}$$

It is quad. in 'c'

Singular Solution: We now

find the singular sol. as,

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Putting in ①, we get

$$4 \cdot \left[\frac{3}{2} (y+c)^{2/3} \right]^2 = 9x$$

$$\Rightarrow 4 \cdot \frac{9}{4} (y+c)^{4/3} = 9x$$

$$\Rightarrow (y+c)^{4/3} = x$$

$$\Rightarrow (y+c)^2 = x^3$$

$$\Rightarrow y^2 + c^2 + 2cy = x^3$$

$$\Rightarrow c^2 + 2yc + y^2 - x^3 = 0 \text{ --- ②}$$

Singular Sol.

We now find the singular sol. as

C-discriminant

C-disc. of ②, is given by

$$B^2 - 4AC = 0$$

$$\Rightarrow (2y)^2 - 4(y^2 - x^3) = 0$$

$$\Rightarrow 4y^2 - 4(y^2 - x^3) = 0$$

$$\Rightarrow x^3 = 0$$

P-discriminant

P-disc. of ①, is given by

$$B^2 - 4AC = 0$$

$$\Rightarrow 0 - 4(4)(-9x) = 0$$

$$\Rightarrow x = 0$$

Common in C-disc., P-disc.

is $x = 0$

But ① is not satisfied by $x = 0$

$\therefore$ no singular sol. of ① exists

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$$P^2 + 2Px^3 - 4x^2y = 0$$

Sol:-

$$4x^2y = P^2 + 2Px^3$$

C-Discriminant:

C-discriminant of ② is given by,

$$B^2 - 4AC = 0$$

$$\Rightarrow (2y^2)^2 - 4(1)[y^2 - x(x-1)] = 0$$

$$\Rightarrow 4y^4 - 4y^2 + 4x(x-1) = 0$$

$$\Rightarrow 4x(x-1) = 0$$

$$\Rightarrow x(x-1) = 0$$

$$\Rightarrow x = 0 \text{ or } (x-1) = 0$$

P-Discriminant:

P-discriminant of ① is given by,

$$B^2 - 4AC = 0$$

$$\Rightarrow 0 - 4(4x)(3x-1)^2 = 0$$

$$\Rightarrow x(3x-1)^2 = 0$$

$$\Rightarrow x = 0 \text{ or } (3x-1)^2 = 0$$

Since $x=0$ is common in both P-disc. and C-disc.

$\therefore x=0$ is not a singular solution of eq. ①.

$\therefore$ It does not satisfy eq. (I).

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$$6P^2y^2 + 3Px - y = 0$$

Sol:-

$$3Px = y - 6P^2y^2$$

$$\Rightarrow 3x = yP^{-1} - 6Py^2$$

Diff. w.r.t y , we get,

$$3 \frac{dx}{dy} = -yP^{-2} \frac{dP}{dy} + P^{-1} - 6(2Py + y^2 \frac{dP}{dy})$$

$$\Rightarrow 3 \cdot \frac{1}{P} = -\frac{y}{P^2} \frac{dP}{dy} + \frac{1}{P} - 12yP - 6y^2 \frac{dP}{dy}$$

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$$4y = P^2x^2 + 2Px$$

Differentiating w.r.t x , we get

$$4 \frac{dy}{dx} = 2x^2 P^2 + 2x^2 P \frac{dP}{dx} + 2(P + x \frac{dP}{dx})$$

$$\Rightarrow 2P = -x^3 P^2 + P + (x^2 P + x) \frac{dP}{dx}$$

$$\Rightarrow (P + \frac{P^2}{x^3}) - (\frac{P}{x^2} + x) \frac{dP}{dx} = 0$$

$$\Rightarrow P(1 + \frac{P}{x^3}) - x(\frac{P}{x^3} + 1) \frac{dP}{dx} = 0$$

$$\Rightarrow (1 + \frac{P}{x^3})(P - x \frac{dP}{dx}) = 0$$

$$\Rightarrow 1 + \frac{P}{x^3} = 0 \text{ or } P - x \frac{dP}{dx} = 0$$

Consider,

$$P - x \frac{dP}{dx} = 0$$

$$\Rightarrow x \frac{dP}{dx} = P$$

$$\Rightarrow \int \frac{dP}{P} = \int \frac{dx}{x}$$

$$\Rightarrow \ln P = \ln x + \ln c$$

$$\Rightarrow P = cx$$

Putting in eq. ①, we get

$$cx^2 + 2cx^4 - 4x^2y = 0 \quad \text{--- ②}$$

Singular Sol.

We now find the S. sol. of ①, as

C-discriminant

C-disc. of ② is given by

$$B^2 - 4AC = 0$$

$$\Rightarrow (2x^4)^2 - 4x^2(-4x^2y) = 0$$

$$\Rightarrow 4x^8 + 16x^4y = 0$$

$$\Rightarrow x^4 + 4y = 0$$

P-discriminant

P-disc. of ① is given by

$$\begin{aligned}\frac{2}{P} + 12yP &= -y\left(\frac{1}{P^2} + 6y\right) \frac{dP}{dy} \\ \Rightarrow 2P\left(\frac{1}{P^2} + 6y\right) + y\left(\frac{1}{P^2} + 6y\right) \frac{dP}{dy} &= 0 \\ \Rightarrow \left(\frac{1}{P^2} + 6y\right) \left(2P + y \frac{dP}{dy}\right) &= 0 \\ \Rightarrow \frac{1}{P^2} + 6y = 0 \text{ or } 2P + y \frac{dP}{dy} &= 0\end{aligned}$$

Consider,

$$\begin{aligned}2P + y \frac{dP}{dy} &= 0 \\ \Rightarrow y \frac{dP}{dy} &= -2P \\ \Rightarrow \int \frac{dP}{P} &= -2 \int \frac{dy}{y} \\ \Rightarrow \ln P &= -2 \ln y + \ln c \\ \Rightarrow P &= c/y^2\end{aligned}$$

putting in ①, we get

$$\begin{aligned}6 \cdot \frac{c}{y^4} \cdot y^2 + 3 \cdot \frac{c}{y^2} \cdot x - y &= 0 \\ \Rightarrow 6c^2 + 3cx - y^3 &= 0 \quad \text{--- ②}\end{aligned}$$

Singular Sol:-

we now find the s.sol. of ①, as

C-discriminant

C-disc. of ②, is given by

$$\begin{aligned}B^2 - 4AC &= 0 \\ \Rightarrow 9x^2 - 4(6)(-y^3) &= 0 \\ \Rightarrow 9x^2 + 24y^3 &= 0 \\ \Rightarrow 3x^2 + 8y^3 &= 0\end{aligned}$$

P-discriminant

P-disc. of ①, is given by

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$$\begin{aligned}B^2 - 4AC &= 0 \\ \Rightarrow (2x^3)^2 - 4(-4x^2y) &= 0 \\ \Rightarrow 4x^6 + 16x^2y &= 0 \\ \Rightarrow x^4 + 4y &= 0\end{aligned}$$

Since c-disc. and P-disc. are same.

$\therefore x^4 + 4y = 0$ is s.sol. of ①

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$$x^3 P^2 + x^2 y P + 1 = 0 \quad \text{--- ①}$$

Sol:-

$$\begin{aligned}x^2 y P &= -x^3 P^2 - 1 \\ \Rightarrow y &= -xP - x^2 P^{-1} \\ \text{Diff. w.r.t } x, \text{ we get}\end{aligned}$$

$$\begin{aligned}\frac{dy}{dx} &= -\left(x \frac{dP}{dx} + P\right) - \left(-x^2 P^{-2} \frac{dP}{dx} - 2x^3 P^{-3}\right) \\ \Rightarrow P &= -x \frac{dP}{dx} - P + \frac{1}{x^2 P^2} \frac{dP}{dx} + \frac{2}{x^3 P} \\ \Rightarrow 2P - \frac{2}{x^3 P} &= -\left(x - \frac{1}{x^2 P^2}\right) \frac{dP}{dx}\end{aligned}$$

$$\Rightarrow 2P\left(1 - \frac{1}{x^3 P^2}\right) + x\left(1 - \frac{1}{x^2 P^2}\right) \frac{dP}{dx} = 0$$

$$\Rightarrow \left(1 - \frac{1}{x^3 P^2}\right) \left(2P + x \frac{dP}{dx}\right) = 0$$

$$\Rightarrow 1 - \frac{1}{x^3 P^2} = 0 \text{ or } 2P + x \frac{dP}{dx} = 0$$

Consider,

$$\begin{aligned}2P + x \frac{dP}{dx} &= 0 \\ \Rightarrow x \frac{dP}{dx} &= -2P\end{aligned}$$

$$\Rightarrow \int \frac{dP}{P} = -2 \int \frac{dx}{x}$$

$$\Rightarrow \ln P = -2 \ln x + \ln c$$

$$B^2 - 4AC = 0$$

$$\Rightarrow 9x^2 - 46y^2(-y) = 0$$

$$\Rightarrow 9x^2 + 24y^3 = 0$$

$$\Rightarrow 3x^2 + 8y^3 = 0$$

Since C-disc. is equal to P-disc. $\therefore 3x^2 + 8y^3 = 0$ is a singular sol. of ①

9 $xP - 2yP^3 + 12x^3 = 0$ ——— ①

Sol:-

$$2yP^3 = xP^4 + 12x^3$$

$$\Rightarrow 2y = xP + 12x^3P^{-3}$$

Diff. it w.r.t x , we get

$$2 \frac{dy}{dx} = x \frac{dP}{dx} + P + 12(-3x^3P^{-4} \frac{dP}{dx} + 3x^2P^{-3})$$

$$\Rightarrow 2P = x \frac{dP}{dx} + P - 36x^3P^{-4} \frac{dP}{dx} + 36x^2P^{-3}$$

$$\Rightarrow P - \frac{36x^2}{P^3} = x \frac{dP}{dx} - \frac{36x^3}{P^4} \frac{dP}{dx}$$

$$\Rightarrow P - \frac{36x^2}{P^3} = x \left(1 - \frac{36x^2}{P^4}\right) \frac{dP}{dx}$$

$$\Rightarrow P \left(1 - \frac{36x^2}{P^4}\right) - x \left(1 - \frac{36x^2}{P^4}\right) \frac{dP}{dx} = 0$$

$$\Rightarrow \left(1 - \frac{36x^2}{P^4}\right) \left(P - x \frac{dP}{dx}\right) = 0$$

$$\Rightarrow 1 - \frac{36x^2}{P^4} = 0 \text{ or } P - x \frac{dP}{dx} = 0$$

Consider,

$$P - x \frac{dP}{dx} = 0$$

$$\Rightarrow x \frac{dP}{dx} = P$$

$$\Rightarrow \int \frac{dP}{P} = \int \frac{dx}{x}$$

$$\Rightarrow \ln P = \ln x^2 + \ln c$$

$$\Rightarrow P = c/x^2$$

putting in eq. ①, we get,

$$x^3 \cdot \frac{c^2}{x^4} + x^2y \cdot \frac{c}{x^2} + 1 = 0$$

$$\Rightarrow c^2x + cxy + x = 0 \text{ ——— ②}$$

Singular sol.

we now find the s. sol. of ② as

C-discriminant

C-disc. of ② is given by

$$B^2 - 4AC = 0$$

$$\Rightarrow x^2y^2 - 4x = 0$$

$$\Rightarrow x(xy^2 - 4) = 0$$

$$\Rightarrow x = 0, xy^2 - 4 = 0$$

P-discriminant

p-disc. of ① is given by

$$B^2 - 4AC = 0$$

$$\Rightarrow x^4y^2 - 4x^3 = 0$$

$$\Rightarrow x^3(xy^2 - 4) = 0$$

$$\Rightarrow x \cdot x^2(xy^2 - 4) = 0$$

$$\Rightarrow x = 0, x^2 = 0, xy^2 - 4 = 0$$

Since $x = 0, xy^2 - 4 = 0$ are

common in both C-disc. and P-disc.

$\therefore x = 0, xy^2 - 4 = 0$ are

the singular sols. of ①

$$\Rightarrow \ln p = \ln x + \ln c$$

$$\Rightarrow p = cx$$

putting in eq. ①, we get,

$$c^4 x^5 - 2c^3 x^3 y + 12x^3 = 0$$

$$\Rightarrow c^4 x^2 - 2c^3 y + 12 = 0 \quad \text{--- ②}$$

Singular Soln.

we now, find singular sol. as

C-discriminant

Since ② is not quadratic

$\therefore$ c-disc. of ② is found as,

Diff. ②, partially w.r.t c, we get

$$4c^3 x^5 - 6c^2 x^3 y = 0$$

$$\Rightarrow 2cx^2 - 3y = 0$$

$$\Rightarrow c = \frac{3y}{2x^2} \text{ put in ②, we get.}$$

$$\left(\frac{3y}{2x^2}\right)^4 x^2 - 2\left(\frac{3y}{2x^2}\right)^3 y + 12 = 0$$

$$\Rightarrow \frac{81y^4}{16x^6} - \frac{27y^4}{4x^6} + 12 = 0$$

$$\Rightarrow 81y^4 - 108y^4 + 192x^6 = 0$$

$$\Rightarrow -27y^4 + 192x^6 = 0$$

$$\Rightarrow -9y^4 + 64x^6 = 0$$

$$\Rightarrow 9y^4 = 64x^6$$

P-discriminant

Diff. ①, partially w.r.t P, we get

$$4xP^3 - 6yP^2 = 0$$

$$\Rightarrow 2xP - 3y = 0$$

$$\Rightarrow P = \frac{3y}{2x} \text{ putting in ①}$$

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$$8P^3 x - 12P^2 y - 27x = 0 \quad \text{--- ①}$$

Sol:

$$12P^2 y = -27x + 8P^3 x$$

$$\Rightarrow 12y = 8Px - 27P^2 x$$

Diff. w.r.t x, we get

$$12 \frac{dy}{dx} = 8(P + x \frac{dP}{dx}) - 27(P^2 + 2xP \frac{dP}{dx})$$

$$\Rightarrow 12P = 8P + 8x \frac{dP}{dx} - \frac{27}{P^2} + \frac{54x}{P^3} \frac{dP}{dx}$$

$$\Rightarrow 4P + \frac{27}{P^2} = 2x(4 + \frac{27}{P^3}) \frac{dP}{dx}$$

$$\Rightarrow P(4 + \frac{27}{P^3}) = 2x(4 + \frac{27}{P^3}) \frac{dP}{dx} = 0$$

$$\Rightarrow (4 + \frac{27}{P^3})(P - 2x \frac{dP}{dx}) = 0$$

$$\Rightarrow 4 + \frac{27}{P^3} = 0 \text{ or } P - 2x \frac{dP}{dx} = 0$$

Consider,

$$P - 2x \frac{dP}{dx} = 0$$

$$\Rightarrow 2x \frac{dP}{dx} = P$$

$$\Rightarrow \int \frac{dP}{P} = \frac{1}{2} \int \frac{dx}{x}$$

$$\Rightarrow \ln P = \frac{1}{2} \ln x + \ln c$$

$$\Rightarrow P = c\sqrt{x}$$

putting in ①, we get

$$8(c\sqrt{x})^3 x - 12(c\sqrt{x})^2 y - 27x = 0$$

$$\Rightarrow 8c^3 x^{3/2} x - 12c^2 x y - 27x = 0$$

$$\Rightarrow 8c^3 x^{5/2} - 12c^2 y - 27 = 0 \quad \text{--- ②}$$

Singular Soln.

we now find the S. sol. of ① as,

$$\begin{aligned}
 & \chi \left(\frac{3y}{2\chi} \right)^4 - 2y \left(\frac{3y}{2\chi} \right)^3 + 12\chi^3 = 0 \\
 \Rightarrow & \frac{81y^4}{16\chi^3} - \frac{27y^4}{4\chi^3} + 12\chi^3 = 0 \\
 \Rightarrow & 81y^4 - 108y^4 + 192\chi^6 = 0 \\
 \Rightarrow & -27y^4 + 192\chi^6 \\
 \Rightarrow & -9y^4 + 64\chi^6 \\
 \Rightarrow & 9y^4 = 64\chi^6
 \end{aligned}$$

Since C-disc. and P-disc. are same

$\therefore 9y^4 = 64\chi^6$ is the S.Sol.

11 Investigate for singular sol. by finding P-disc.

$$e^P - P + \chi y - \chi - 1 = 0 \quad \text{--- ①}$$

Sol:-

P-discriminant

Diff. ①, partially w.r.t P, we get.

$$\begin{aligned}
 e^P - 1 &= 0 \\
 \Rightarrow e^P &= 1 \\
 \Rightarrow \ln e^P &= \ln 1 \\
 \Rightarrow P \ln e &= \ln 1 \\
 \Rightarrow P &= 0 \quad (\because \ln e = 1, \ln 1 = 0) \\
 \text{putting in eq. ①, we get} \\
 \chi y - \chi &= 0 \\
 \Rightarrow \chi(y-1) &= 0 \\
 \Rightarrow \chi &= 0, \quad y-1=0
 \end{aligned}$$

Take $\chi = 0$

$$\begin{aligned}
 \text{Since } \chi &= 0 \quad \therefore \frac{d\chi}{dy} = 0 \\
 \text{or } \frac{1}{P} &= 0
 \end{aligned}$$

Since eq. ① is not satisfied

C-discriminant

Diff. ② partially w.r.t C, we get

$$\begin{aligned}
 24C^2 \chi^{3/2} - 24Cy &= 0 \\
 \Rightarrow C\chi^{3/2} - y &= 0 \\
 \Rightarrow C &= \frac{y}{\chi^{3/2}} \text{ putting in ②, we get}
 \end{aligned}$$

$$\begin{aligned}
 8 \left(\frac{y}{\chi^{3/2}} \right)^3 \chi^{3/2} - 12 \left(\frac{y}{\chi^{3/2}} \right)^2 y - 27 &= 0 \\
 \Rightarrow 8 \frac{y^3}{\chi^3} - 12 \frac{y^3}{\chi^3} - 27 &= 0 \\
 \Rightarrow 8y^3 - 12y^3 - 27\chi^3 &= 0 \\
 \Rightarrow -4y^3 - 27\chi^3 &= 0 \\
 \Rightarrow 27\chi^3 + 4y^3 &= 0
 \end{aligned}$$

P-discriminant

Diff. ① partially w.r.t P, we get

$$\begin{aligned}
 24P^2 \chi - 24Py &= 0 \\
 \Rightarrow P\chi &= y \\
 \Rightarrow P &= y/\chi \text{ putting in ① we get} \\
 8 \frac{y^3}{\chi^3} \cdot \chi - 12 \frac{y^2}{\chi^2} \cdot y - 27\chi &= 0 \\
 \Rightarrow 8 \frac{y^3}{\chi^2} - 12 \frac{y^3}{\chi^2} - 27\chi &= 0 \\
 \Rightarrow 8y^3 - 12y^3 - 27\chi^3 &= 0 \\
 \Rightarrow -4y^3 - 27\chi^3 &= 0 \\
 \Rightarrow 27\chi^3 + 4y^3 &= 0
 \end{aligned}$$

Since C-disc. and P-disc. are same

$\therefore 27\chi^3 + 4y^3 = 0$ is the S.Sol. of ①

12 Investigate for singular sol. by finding P-disc.

$$4\chi(\chi-1)(\chi-2)P^2 - (3\chi^2 - 6\chi + 2)^2 = 0$$

by $x=0$ and $\frac{1}{p}=0$

$\therefore x=0$ is not singular sol. of ①

Take $y-1=0$

$$y=1 \text{ and } \frac{dy}{dx}=0$$

$$\text{or } p=0$$

Since ① is satisfied by $y=1$
and $p=0$

$\therefore y-1=0$ is a S. sol. of ①

13 $P^4 - 2P^2 + 1 - y^4 = 0$ — ①

Sol:-

P-discriminant

Diff. ① partially w.r.t P, we get

$$4P^3 - 4P = 0$$

$$\Rightarrow 4P(P^2 - 1) = 0$$

$$\Rightarrow P(P-1)(P+1) = 0$$

$$\Rightarrow P=0, P=1, P=-1$$

Putting in eq. ①, we get

$$1 - y^4 = 0$$

$$, y^4 = 0$$

$$\text{or } (1-y^2)(1+y^2) = 0$$

$$, y = 0$$

$$\text{or } (1-y)(1+y)(1+y^2) = 0$$

$$, y = 0$$

$$0 \quad 1-y=0, 1+y=0, y=0$$

$$\left(\begin{array}{l} 1+y^2=0 \text{ is rejected} \\ \because y \text{ is imaginary} \end{array} \right)$$

Take $1-y=0$

$$y=1 \quad \therefore \frac{dy}{dx}=0 \quad \text{i.e. } p=0$$

Since ① is satisfied by

$$y=1 \text{ and } \frac{dy}{dx}=0$$

Sol:-

P-discriminant

P-disc of ① is given by

$$B^2 - 4AC = 0$$

$$\Rightarrow 0 - 4 \cdot 4x(x-1)(x-2) \cdot [-(3x^2-6x+2)] = 0$$

$$\Rightarrow x(x-1)(x-2)(3x^2-6x+2) = 0$$

$$\Rightarrow x=0, x-1=0, x-2=0$$

$$3x^2-6x+2=0$$

First, we write ① as

$$4x(x-1)(x-2) = (3x^2-6x+2)^2 \cdot \frac{1}{p^2} \quad \text{--- ②}$$

Take $x=0$

$$x=0 \quad \therefore \frac{dx}{dy}=0 \quad \text{i.e. } \frac{1}{p}=0$$

Since ② is satisfied by

$$x=0 \text{ and } \frac{dx}{dy}=0$$

$\therefore x=0$ is a S. sol. of ①

Take $x-1=0$

$$x=1 \quad \therefore \frac{dx}{dy}=0 \quad \text{i.e. } \frac{1}{p}=0$$

Since ② is satisfied by

$$x=1 \text{ and } \frac{dx}{dy}=0$$

$\therefore x-1=0$ is a S. sol. of ①

Take $x-2=0$

$$x=2 \quad \therefore \frac{dx}{dy}=0 \quad \text{i.e. } \frac{1}{p}=0$$

Since ② is satisfied by

$$x=2 \text{ and } \frac{dx}{dy}=0$$

$\therefore x-2=0$ is a S. sol. of ①

$y = -1$ is S.Sol. of ①

Take $1+y=0$

$$y = -1 \therefore \frac{dy}{dx} = 0 \text{ i.e. } P = 0$$

Since ① is satisfied by

$$y = -1 \text{ and } \frac{dy}{dx} = 0$$

$\therefore y = -1$ is S.Sol. of ①

Take $y = 0$

$$y = 0 \therefore \frac{dy}{dx} = 0 \text{ i.e. } P = 0$$

Since ① is not satisfied by

$$y = 0 \text{ and } \frac{dy}{dx} = 0$$

$\therefore y = 0$ is not a S.Sol. of ①

15 $4P^3 + 3XP - Y = 0$ ——— ①

Sol:-

P-discriminant

Diff. ①, partially wr.t P, we get

$$12P^2 + 3X = 0$$

$$\Rightarrow 4P^2 = -X$$

$$\Rightarrow P^2 = -\frac{X}{4}$$

$$\Rightarrow P = \pm \sqrt{-\frac{X}{4}}$$

putting in eq. ①, we get

$$4\left(\pm \left(-\frac{X}{4}\right)^{\frac{3}{2}}\right) + 3X\left(\pm \left(-\frac{X}{4}\right)^{\frac{1}{2}}\right) - Y = 0$$

$$\Rightarrow \pm \left(-\frac{X}{4}\right)^{\frac{3}{2}} (4\left(-\frac{X}{4}\right) + 3X) - Y = 0$$

$$\Rightarrow \pm \left(-\frac{X}{4}\right)^{\frac{3}{2}} (-2X) - Y = 0$$

$$\Rightarrow Y = \pm 2\left(-\frac{X}{4}\right)^{\frac{3}{2}} X$$

$$\Rightarrow Y^2 = 4\left(-\frac{X}{4}\right) \cdot X^2$$

$$\Rightarrow Y^2 = -X^3$$

$$\Rightarrow X^3 + Y^2 = 0$$

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Take $3X^2 - 6X + 2 = 0$

$$X = \frac{6 \pm \sqrt{36 - 24}}{6} = \frac{6 \pm \sqrt{12}}{6} = \frac{3 \pm \sqrt{3}}{3}$$

Thus $X = 1 \pm \frac{1}{\sqrt{3}} \therefore \frac{dX}{dY} = 0$ i.e. $Y_P = 0$

Since ② is not satisfied

by $X = 1 \pm \frac{1}{\sqrt{3}}$ and $\frac{dX}{dY} = 0$

$3X^2 - 6X + 2 = 0$ is not S.Sol. of ①

14 $P^3 - 4XP + 8Y^2 = 0$ ——— ①

Sol:-

P-discriminant

Diff. ① partially wr.t P, we get

$$3P^2 - 4XY = 0$$

$$\Rightarrow 3P^2 = 4XY$$

$$\Rightarrow P^2 = \frac{4}{3}XY$$

$$\Rightarrow P = \pm \left(\frac{4}{3}XY\right)^{\frac{1}{2}}$$

putting in eq. ①, we get,

$$\pm \left(\frac{4}{3}XY\right)^{\frac{3}{2}} - 4XY\left[\pm \left(\frac{4}{3}XY\right)^{\frac{1}{2}}\right] + 8Y^2 = 0$$

$$\Rightarrow \pm \left(\frac{4}{3}XY\right)^{\frac{3}{2}} \left[\frac{4}{3}XY - 4XY\right] + 8Y^2 = 0$$

$$\Rightarrow \pm \left(\frac{4}{3}XY\right)^{\frac{3}{2}} \left(-\frac{8}{3}XY\right) + 8Y^2 = 0$$

$$\Rightarrow -2\left[\pm \left(\frac{4}{3}XY\right)^{\frac{3}{2}} \cdot \left(\frac{4}{3}XY\right)\right] + 8Y^2 = 0$$

$$\Rightarrow \pm \left(\frac{4}{3}XY\right)^{\frac{3}{2}} - 4Y^2 = 0$$

$$\Rightarrow \pm \left(\frac{4}{3}XY\right)^{\frac{3}{2}} = 4Y^2$$

$$\Rightarrow \left(\frac{4}{3}XY\right)^3 = 16Y^4$$

$$\Rightarrow \frac{64}{27}X^3Y^3 - 16Y^4 = 0$$

$$\Rightarrow \frac{4}{27}X^3Y^3 - Y^4 = 0$$

Take $x^3 + y^2 = 0$

$$y^2 = -x^3$$

$$\therefore 2y \frac{dy}{dx} = -3x^2$$

$$\Rightarrow 2yP = -3x^2$$

$$\Rightarrow P = -\frac{3x^2}{2y}$$

Putting in L.H.S of eq. ①, we get

L.H.S of eq. ①

$$= 4P^3 + 3xP - y$$

$$= 4\left(-\frac{3x^2}{2y}\right)^3 + 3x\left(-\frac{3x^2}{2y}\right) - y$$

$$= 4\left(-\frac{27x^6}{8y^3}\right) - \frac{9x^3}{2y} - y$$

$$= -\frac{27x^6}{2y^3} - \frac{9x^3}{2y} - y$$

$$= -\frac{27(x^3)^2}{2y^3} - \frac{9x^3}{2y} - y$$

$$= -\frac{27(-y^2)^2}{2y^3} - \frac{9(-y^2)}{2y} - y \quad \because x^3 = -y^2$$

$$= -\frac{27y^4}{2y^3} + \frac{9y^2}{2y} - y$$

$$= -\frac{27y}{2} + \frac{9y}{2} - y$$

$$= \frac{-27y + 9y - 2y}{2} = \frac{-20y}{2} = -10y \neq 0 = R.H.S$$

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$$P^3 - 2x^2P - 4xy = 0 \quad \text{--- ①}$$

Sol.:

P-discriminant

Diff. ① partially w.r.t P, we get

$$3P^2 - 2x^2 = 0$$

$$\Rightarrow 3P^2 = 2x^2$$

$$\Rightarrow P = \pm \left(\frac{2}{3}\right)^{\frac{1}{2}} x$$

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$$4x^3y^3 - 27y^4 = 0$$

$$\Rightarrow y^3(4x^3 - 27y) = 0$$

$$\Rightarrow y^3 = 0 \quad , \quad 4x^3 - 27y = 0$$

$$\Rightarrow y = 0 \quad , \quad 4x^3 - 27y = 0$$

Take $y = 0$

$$y = 0 \therefore \frac{dy}{dx} = 0 \quad \text{i.e. } P = 0$$

Since ① is satisfied by

$$y = 0 \quad \text{and} \quad \frac{dy}{dx} = 0$$

$\therefore y = 0$ is a s.sol. of ①

Take $4x^3 - 27y = 0$

$$4x^3 - 27y = 0$$

$$\Rightarrow 27y = 4x^3$$

$$\Rightarrow y = \frac{4}{27}x^3 \therefore \frac{dy}{dx} = \frac{4}{9}x^2$$

$$\text{or } P = \frac{4}{9}x^2$$

putting in L.H.S of eq. ①

L.H.S. of eq. ①

$$= P^3 - 4xyP + 8y^2$$

$$= \left(\frac{4}{9}x^2\right)^3 - 4x\left(\frac{4}{27}x^3\right)\left(\frac{4}{9}x^2\right) + 8\left(\frac{4}{27}x^3\right)^2$$

$$= \frac{64}{729}x^6 - \frac{64}{243}x^6 + \frac{128}{729}x^6$$

$$= \frac{64x^6 - 192x^6 + 128x^6}{729} = 0 = R.H.S$$

Since eq. ① is satisfied by

$$y = \frac{4}{27}x^3 \quad \text{and} \quad \frac{dy}{dx} = \frac{4}{9}x^2$$

$\therefore 4x^3 - 27y = 0$ is a s.sol. of ①

putting in eq. ①, we get

$$\left(\pm\left(\frac{2}{3}\right)^{\frac{1}{2}}x\right)^3 - 2x^2\left(\pm\left(\frac{2}{3}\right)^{\frac{1}{2}}x\right) - 4xy = 0$$

$$\Rightarrow \pm\left(\frac{2}{3}\right)^{\frac{3}{2}}x^3 - 2\left(\pm\left(\frac{2}{3}\right)^{\frac{1}{2}}x^3\right) - 4xy = 0$$

$$\Rightarrow \pm\left(\frac{2}{3}\right)^{\frac{1}{2}}x^3\left(\frac{2}{3} - 2\right) - 4xy = 0$$

$$\Rightarrow \pm\left(\frac{2}{3}\right)^{\frac{1}{2}}x^3\left(-\frac{4}{3}\right) = 4xy$$

$$\Rightarrow \pm\left(\frac{2}{3}\right)^{\frac{1}{2}}x^3 = -3xy$$

$$\Rightarrow \frac{2}{3}x^6 = 9xy^2$$

$$\Rightarrow 2x^6 - 27x^2y^2 = 0$$

$$\Rightarrow x^2(2x^4 - 27y^2) = 0$$

$$\Rightarrow x^2 = 0, \quad 2x^4 - 27y^2 = 0$$

$$\Rightarrow x = 0 \quad 2x^4 - 27y^2 = 0$$

Take $x = 0$

$$x = 0 \therefore \frac{dx}{dy} = 0 \quad \text{i.e. } \frac{1}{p} = 0$$

first, we write ①, as,

$$1 - 2x^2\left(\frac{1}{p}\right)^2 - 4xy\left(\frac{1}{p}\right)^3 = 0 \quad \text{--- ②}$$

since ②, is not satisfied

by $x = 0$ and $\frac{dx}{dy} = 0$

$\therefore x = 0$ is not a s. sol. of ①

Take $2x^4 - 27y^2 = 0$

$$2x^4 - 27y^2 = 0$$

$$\Rightarrow 27y^2 = 2x^4$$

$$\Rightarrow y^2 = \frac{2}{27}x^4$$

$$\Rightarrow y = \pm\left(\frac{2}{27}\right)^{\frac{1}{2}}x^2$$

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$$\therefore \frac{dy}{dx} = \pm 2\left(\frac{2}{27}\right)^{\frac{1}{2}}x$$

$$\Rightarrow p = \pm 2\left(\frac{2}{27}\right)^{\frac{1}{2}}x$$

putting in L.H.S of eq. ①

L.H.S of eq. ①

$$= p^3 - 2x^2p - 4xy$$

$$= \left[\pm 2\left(\frac{2}{27}\right)^{\frac{1}{2}}x\right]^3 - 2x^2\left[\pm 2\left(\frac{2}{27}\right)^{\frac{1}{2}}x\right] - 4x\left[\pm\left(\frac{2}{27}\right)^{\frac{1}{2}}x^2\right]$$

$$= \pm 8\left(\frac{2}{27}\right)^{\frac{3}{2}}x^3 - \left[\pm 4\left(\frac{2}{27}\right)^{\frac{1}{2}}x^3\right] - \left[\pm 4\left(\frac{2}{27}\right)^{\frac{1}{2}}x^3\right]$$

$$= \pm 4x^3\left(\frac{2}{27}\right)^{\frac{1}{2}}\left[2\left(\frac{2}{27}\right) - 1 - 1\right]$$

THE END.