

(125) $4.5 + 4.6$

$$\Rightarrow \frac{1}{\sqrt{x} + 2x^{1/3}} \Rightarrow \int \frac{1}{t^3 + 2t^2} 6t^5 dt$$

$$\int \frac{6t^5}{t^3 + 2t^2} dt \Rightarrow 6 \int \frac{t^5}{t^2(t+2)} dt$$

$$= 6 \int \frac{t^3}{t+2} dt$$

$$= 6 \int \frac{t^2 - 2t + 4}{t+2} dt + \int \frac{-8}{t+2} dt \cdot t+2 \left| \begin{array}{l} t^2 - 2t + 4 \\ -t^3 + 2t^2 \end{array} \right.$$

$$6 \left[\frac{t^3}{3} - \frac{2t^2}{2} + 4t - 8 \int \frac{1}{t+2} dt \right] \quad \begin{array}{l} -2t^2 \\ +2t^2 + 4t \end{array}$$

$$6 \left[\frac{t^3}{3} - t^2 + 4t - 8 \ln|t+2| \right] + C \quad \begin{array}{l} 4t \\ -4t + 8 \end{array}$$

$$= 6 \left[\frac{x^{3/6}}{3} - x^{2/6} + 4x^{1/6} - 8 \ln|x^{1/6} + 2| \right] + C \quad -8$$

$$= \left[6 \frac{x^{3/6}}{3} - 6x^{2/6} + 24x^{1/6} - 48 \ln|x^{1/6} + 2| \right] + C$$

$$= 2x^{3/6} - 6x^{2/6} + 24x^{1/6} - 48 \ln|x^{1/6} + 2| + C \text{ Ans.}$$

Reduction formula.

$$\int \sin^n x dx = -\cos x \frac{\sin^{n-1} x}{n} + \frac{n-1}{n} \int \sin^{n-2} x dx$$

where n.

$$\int \cos^n x dx = \sin x \frac{\cos^{n-1} x}{n} + \frac{n-1}{n} \int \cos^{n-2} x dx \text{ an integ}$$

Exercise NO 4.6

(1) $\int \sin^5 x dx$

$$= \int \sin^4 x \sin x dx$$

(126) 4.6

$$\int (\sin^2 x)^2 \sin x \, dx$$

$$\int (1 - \cos^2 x)^2 \sin x \, dx$$

put $\cos x = t$

$$-\sin x \, dx = dt$$

$$\sin x \, dx = -dt$$

$$\int (1 - t^2)^2 (-dt)$$

$$-\int (1 + t^4 - 2t^2) \, dt$$

$$= -t - \frac{t^5}{5} + 2\frac{t^3}{3} + C$$

$$= -\cos x - \frac{(\cos x)^5}{5} + \frac{2}{3}(\cos x)^3 + C$$

$$= -\left(\cos x + \frac{1}{5}\cos^5 x - \frac{2}{3}\cos^3 x\right) + C \text{ Ans.}$$

2

$$\int \cos^6 x \, dx$$

$$\int \cos^4 x \cdot \cos x \, dx$$

$$\int (\cos^2 x)^2 \cos x \, dx$$

$$= \int (1 - \sin^2 x)^2 \cos x \, dx$$

put $\sin x = t$

$$\cos x \, dx = dt$$

$$\int (1 - t^2)^2 \, dt$$

$$= \int (1 - t^2 - 3(1)(t^2)(1 - t^2)) \, dt$$

(127)

4.6

$$\int (1 - t^6 - 3t^2 + 3t^4) dt$$

$$= t - \frac{t^7}{7} - \frac{3t^3}{3} + \frac{3t^5}{5} + c$$

$$= \sin x - \frac{\sin^7 x}{7} - \sin^3 x + \frac{3}{5} \sin^5 x + c$$

$$= \sin x - \sin^3 x + \frac{3}{5} \sin^5 x - \frac{1}{7} \sin^7 x + c \text{ Ans.}$$

3

$$\int \sin^8 x dx$$

by using reduction formula.

$$= \cos x \sin^{n-1} x + \frac{n-1}{n} \int \sin^{n-2} x dx$$

$$= -\cos x \sin^{8-1} x + \frac{(8-1)}{8} \int \sin^{8-2} x dx$$

$$\int \sin^8 x dx = -\cos x \sin^7 x + \frac{7}{8} \int \sin^6 x dx$$

$$= -\cos x \sin^7 x + \frac{7}{8} \left[-\cos x \sin^5 x + \frac{5}{6} \int \sin^4 x dx \right]$$

$$= \text{" " " " } - \frac{7}{48} \cos x \sin^5 x + \frac{35}{48} \int \sin^4 x dx$$

$$= \text{" " " " " " } + \frac{35}{48} \left[\frac{-\cos x \sin^3 x}{4} + \frac{3}{4} \int \sin^2 x dx \right]$$

$$\text{" " " " " " " " } - \frac{35}{192} \cos x \sin^3 x + \frac{105}{192} \int \sin^2 x dx$$

$$\text{" " " " " " " " " " } + \frac{105}{192} \int \left(\frac{1 - \cos 2x}{2} \right) dx$$

$$\text{" " " " " " " " " " " " } + \frac{105}{192} \left[\frac{1}{2} \int dx - \frac{1}{2} \int \cos 2x dx \right]$$

$$\text{" " " " " " " " " " " " " " } \frac{35}{384} x - \frac{35}{384} \frac{\sin 2x}{2} + c$$

$$= -\frac{\cos x \sin^7 x}{8} - \frac{7}{48} \cos x \sin^5 x - \frac{35}{192} \cos x \sin^3 x + \frac{35}{128} x$$

$$- \frac{35}{128} \frac{2 \sin x \cos x}{2} + C$$

$$= -\frac{\cos x \sin^7 x}{8} - \frac{7}{48} \cos x \sin^5 x - \frac{35}{192} \cos x \sin^3 x + \frac{35}{128} x - \frac{35}{128} \cos x \sin x + C \text{ Ans.}$$

4

$$\int \cos^6 x \, dx$$

by using reduction formula.

$$= \frac{\sin x \cos^{n-1} x}{n} + \frac{n-1}{n} \int \cos^{n-2} x \, dx$$

$$\frac{\sin x \cos^5 x}{6} + \frac{5}{6} \int \cos^4 x \, dx$$

$$\frac{\sin x \cos^5 x}{6} + \frac{5}{6} \left[\frac{\sin x \cos^3 x}{4} + \frac{3}{4} \int \cos^2 x \, dx \right]$$

$$= \frac{\sin x \cos^5 x}{6} + \frac{5}{24} \sin x \cos^3 x + \frac{15}{24} \int \cos^2 x \, dx$$

$$\frac{\sin x \cos^5 x}{6} + \frac{5}{24} \sin x \cos^3 x + \frac{15}{24} \int \left(\frac{1 + \cos 2x}{2} \right) dx$$

$$+ \frac{15}{24} \left[\frac{1}{2} \int 1 \, dx + \frac{1}{2} \int \cos 2x \, dx \right]$$

$$+ \frac{15}{24} \left[\frac{1}{2} x + \frac{1}{2} \frac{\sin 2x}{2} \right]$$

$$= \frac{5}{48} x + \frac{5}{48} x \sin x \cos x + C$$

$$= \frac{5}{16} \sin x \cos x + \frac{5}{16} x + C$$

$$\int \cos^6 x \, dx = \frac{\sin x \cos^5 x}{6} + \frac{5}{24} \sin x \cos^3 x + \frac{5}{16} \sin x \cos x + \frac{5}{16} x + C \text{ Ans.}$$

(129) 4.6

5

$$\int \tan^n x \, dx$$

$$I = \int \tan^{n-2+2} x \, dx$$

$$= \int \tan^{n-2} x \cdot \tan^2 x \, dx$$

$$= \int \tan^{n-2} x (\sec^2 x - 1) \, dx$$

$$\int \tan^{n-2} x \sec^2 x \, dx - \int \tan^{n-2} x \, dx$$

$$= \frac{\tan^{n-2+1} x}{n-2+1} - \int \tan^{n-2} x \, dx$$

$$= \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x \, dx$$

$$\int \tan^n x \, dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x \, dx$$

6

$$\int \sec^n x \, dx$$

$$I = \int \sec^{n-2+2} x \, dx$$

$$\int \sec^{n-2} x \cdot \sec^2 x \, dx$$

$$= \sec^{n-2} x \int \sec^2 x \, dx - \int \frac{d}{dx} (\sec^{n-2} x) \int \sec^2 x \, dx$$

$$\sec^{n-2} x \tan x - \int (n-2) \sec^{n-3} x (\sec x \tan x) \cdot \tan x \, dx$$

$$= (n-2) \int \sec^{n-2} x (\sec^2 x - 1) \, dx$$

$$= (n-2) \int \sec^n x \, dx + (n-2) \int \sec^{n-2} x \, dx$$

$$\sec^{n-2} x \tan x - (n-2)I + (n-2) \int \sec^{n-2} x dx$$

$$I + I(n-2) = \sec^{n-2} x \tan x + (n-2) \int \sec^{n-2} x dx$$

$$I(n-1) = \sec^{n-2} x \tan x + (n-2) \int \sec^{n-2} x dx$$

$$= \frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1} \int \sec^{n-2} x dx$$

$$\frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1} \int \sec^{n-2} x dx \text{ Ans.}$$

7 $\int \cot^n x dx$

$$I = \int \cot^n x dx$$

$$I = \int \cot^{n-2} x \cot^2 x dx$$

$$\int \cot^{n-2} x \cot^2 x dx \Rightarrow \int \cot^{n-2} x (\operatorname{cosec}^2 x - 1) dx$$

$$= \int \cot^{n-2} x \operatorname{cosec}^2 x dx - \int \cot^{n-2} x dx$$

$$= \frac{\cot^{n-1} x}{n-1} - \int \cot^{n-2} x dx$$

$$= -\frac{\cot^{n-1} x}{n-1} - \int \cot^{n-2} x dx \text{ Ans.}$$

(8) $\int \operatorname{cosec}^n x dx$

$$I = \int \operatorname{cosec}^{n-2} x \operatorname{cosec}^2 x dx$$

$$= \int \operatorname{cosec}^{n-2} x \cdot \operatorname{cosec}^2 x dx$$

$$\int \csc^{n-2} x \csc^2 x dx = \int \left[\frac{d}{dx} (\csc^{n-2} x) \right] \csc^2 x dx$$

$$\csc^{n-2} x (-\cot x) - \int (n-2) \csc^{n-3} x (-\csc x \cot x) \cot x dx$$

$$= -\csc^{n-2} x \cot x + (n-2) \int \csc^{n-2} x \cot^2 x dx$$

$$= -\csc^{n-2} x \cot x + (n-2) \int \csc^{n-2} x (\csc^2 x - 1) dx$$

$$\text{Ans.} \quad = -\csc^{n-2} x \cot x + (n-2) \int \csc^{n-2} x dx$$

$$= -\csc^{n-2} x \cot x + (n-2) I$$

$$I + (n-2)I = -\csc^{n-2} x \cot x + (n-2) \int \csc^{n-2} x dx$$

$$I(n-1) = -\csc^{n-2} x \cot x + (n-2) \int \csc^{n-2} x dx$$

$$\int \csc^{n-2} x dx = \frac{-\csc^{n-2} x \cot x}{n-1} + \frac{(n-2)}{(n-1)} \int \csc^{n-2} x dx$$

9

$$\int \tan^6 x dx$$

$$\int \tan^n x dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x dx$$

$$= \frac{\tan^5 x}{5} - \int \tan^4 x dx$$

$$\frac{\tan^5 x}{5} - \left[\frac{\tan^3 x}{3} + \int \tan^2 x dx \right]$$

$$\frac{\tan^5 x}{5} - \frac{\tan^3 x}{3} + \int (\sec^2 x - 1) dx$$

$$\frac{\tan^5 x}{5} - \frac{\tan^3 x}{3} + \tan x - x + C$$

(20)

$$\int \cot^5 x \, dx$$

$$\int \cot^n x \, dx = -\frac{\cot^{n-1} x}{n-1} - \int \cot^{n-2} x \, dx$$

$$\int \cot^5 x \, dx = -\frac{\cot^4 x}{4} - \int \cot^3 x \, dx$$

$$= -\frac{\cot^4 x}{4} - \left[-\frac{\cot^2 x}{2} - \int \cot x \, dx \right]$$

$$= -\frac{\cot^4 x}{4} + \frac{\cot^2 x}{2} + \int \cot x \, dx$$

$$= -\frac{\cot^4 x}{4} + \frac{\cot^2 x}{2} + \int \frac{\cos x}{\sin x} \, dx$$

$$= -\frac{\cot^4 x}{4} + \frac{\cot^2 x}{2} + \ln |\sin x| + c \text{ Ans.}$$

11

$$\int \sec^6 x \, dx$$

$$\int \sec^n x \, dx = \frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1} \int \sec^{n-2} x \, dx$$

$$\int \sec^6 x \, dx = \frac{\sec^4 x \tan x}{5} + \frac{4}{5} \int \sec^4 x \, dx$$

$$= \frac{\sec^4 x \tan x}{5} + \frac{4}{5} \left[\frac{\sec^2 x \tan x}{3} + \frac{2}{3} \int \sec^2 x \, dx \right]$$

$$= \frac{\sec^4 x \tan x}{5} + \frac{4}{15} \sec^2 x \tan x + \frac{8}{15} \tan x + c$$

$$= \frac{\sec^4 x \tan x}{5} + \frac{4}{15} \sec^2 x \tan x + \frac{8}{15} \tan x + c \text{ Ans.}$$

12

$$\int \operatorname{Cosec}^5 x \, dx$$

$$\operatorname{Cosec}^n x \, dx = -$$

$$\operatorname{Cosec}^5 x \, dx = -\operatorname{Cosec}^3 x \cot x + \frac{3}{4} \int \operatorname{Cosec}^3 x \, dx$$

$$= -\operatorname{Cosec}^3 x \cot x + \frac{3}{4} \left[-\operatorname{Cosec} x \cot x + \frac{1}{2} \int \operatorname{Cosec} x \, dx \right]$$

$$= -\operatorname{Cosec}^3 x \cot x - \frac{3}{8} \operatorname{Cosec} x \cot x + \frac{3}{8} \ln |\operatorname{Cosec} x - \cot x|$$

$$= \dots + \frac{3}{8} \ln |\tan(x/2)| + c$$

$$= -\operatorname{Cosec}^3 x \cot x - \frac{3}{8} \operatorname{Cosec} x \cot x + \frac{3}{8} \ln |\tan(x/2)| + c \text{ Ans.}$$

Useful Substitutions (only Trigonometric functions)

$$dz = \frac{2dz}{1+z^2}$$

$$\cos x = \frac{1-z^2}{1+z^2}$$

$$\sin x = \frac{2z}{1+z^2}$$

$$\tan x = \frac{2z}{1-z^2}$$

Suppose:-

$$z = \tan(x/2)$$

$$dz = \sec^2(x/2) \cdot \frac{1}{2} dx$$

$$dx = \frac{2dz}{\sec^2(x/2)}$$

$$dx = \frac{2dz}{1+\tan^2(x/2)}$$

$$dx = \frac{2dz}{1+z^2}$$

$$\sin x = 2 \sin x/2 \cos x/2$$

$$\dots = 2 \sin x/2 \cos x/2$$

(134) 4.6

Dividing by $\cos^2 x/2$

$$\sin x = \frac{2 \sin x/2 / \cos x/2}{1 + \tan^2 x/2}$$

$$\sin x = \frac{2 \tan x/2}{1 + \tan^2 x/2}$$

$$\sin x = \frac{2z}{1+z^2}$$

$$\because \tan x/2 = z$$

$$\cos x = \frac{\cos^2 x/2 - \sin^2 x/2}{\cos^2 x/2 + \sin^2 x/2}$$

$$\cos x = \frac{\cos^2 x/2 - \sin^2 x/2}{\cos^2 x/2 + \sin^2 x/2}$$

Dividing by $\cos^2 x/2$ in R.H.S.

$$= \frac{1 - \tan^2 x/2}{1 + \tan^2 x/2}$$

$$\cos x = \frac{1 - z^2}{1 + z^2}$$

Now

$$\tan x = \frac{2 \tan x/2}{1 - \tan^2 x/2}$$

$$\tan x = \frac{2z}{1 - z^2}$$

13

$$\int \frac{dx}{a + b \sin x}$$

$$z = \tan x/2$$

$$dx = \frac{2dz}{1+z^2}$$

$$\sin x = \frac{2z}{1+z^2}$$

$$I = \int \frac{2dz/(1+z^2)}{a + b(2z/(1+z^2))}$$

$$I = \int \frac{2dz/\sqrt{1+z^2}}{a(1+z^2) + 2bz/\sqrt{1+z^2}}$$

$$= \int \frac{2dz}{a + az^2 + 2bz}$$

$$\frac{2}{a} \int \frac{dz}{z^2 + \frac{2b}{a}z + \left(\frac{b}{a}\right)^2 - \left(\frac{b}{a}\right)^2 + 1}$$

$$\frac{2}{a} \int \frac{dz}{\left(z + \frac{b}{a}\right)^2 + \frac{a^2 - b^2}{a^2}}$$

Case I:- if $a > b$

$\because a^2 - b^2$ is true.

$$\frac{2}{a} \int \frac{dz}{\left(z + \frac{b}{a}\right)^2 + \left(\frac{\sqrt{a^2 - b^2}}{a}\right)^2}$$

$$\frac{2}{a} \cdot \frac{1}{\frac{\sqrt{a^2 - b^2}}{a}} \tan^{-1} \left(\frac{z + \frac{b}{a}}{\frac{\sqrt{a^2 - b^2}}{a}} \right) + C$$

$$\frac{2}{a} \times \frac{a}{\sqrt{a^2 - b^2}} \tan^{-1} \left(\frac{az + b/a}{\sqrt{a^2 - b^2}/a} \right)$$

$$\frac{2}{\sqrt{a^2 - b^2}} \tan^{-1} \left(\frac{az + b}{\sqrt{a^2 - b^2}} \right) + C$$

$$\frac{2}{\sqrt{a^2 - b^2}} \tan^{-1} \left(\frac{a \tan x/2 + b}{\sqrt{a^2 - b^2}} \right) + C$$

Case II if $a < b$

$\because b^2 - a^2$ is true

$$\frac{2}{a} \int \frac{dz}{\left(z + \frac{b}{a}\right)^2 - \left(\frac{b^2 - a^2}{a^2}\right)}$$

$$\frac{2}{a} \int \frac{dz}{\left(z + \frac{b}{a}\right)^2 - \left(\frac{\sqrt{b^2 - a^2}}{a}\right)^2}$$

(136) 4.6

$$\therefore \int \frac{1}{x^2 - a^2} = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right|$$

$$\frac{z}{a} \cdot \frac{1}{2\sqrt{b^2 - a^2}} \ln \left| \frac{z + b/a - \sqrt{b^2 - a^2}/a}{z + b/a + \sqrt{b^2 - a^2}/a} \right| + C$$

$$\frac{1}{\sqrt{b^2 - a^2}} \ln \left| \frac{az + b - \sqrt{b^2 - a^2}}{az + b + \sqrt{b^2 - a^2}} \right| + C$$

$$\frac{1}{\sqrt{b^2 - a^2}} \ln \left| \frac{a \tan x/2 + b - \sqrt{b^2 - a^2}}{a \tan x/2 + b + \sqrt{b^2 - a^2}} \right| + C$$

Useful Substitutions:-

(In case of hyperbolic function)

$$z = \tanh(x/2)$$

$$dz = \operatorname{sech}^2(x/2) \cdot \frac{1}{2} dx$$

$$dx = \frac{2 dz}{\operatorname{sech}^2(x/2)}$$

$$dx = \frac{2 dz}{1 - \tanh^2(x/2)}$$

$$dz = \frac{2 dz}{1 - z^2}$$

Now

$$\sinh x = \frac{2 \sinh x/2 \cosh x/2}{\cosh^2 x/2 - \sinh^2 x/2}$$

Dividing by $\cosh^2 x/2$

$$= \frac{2 \tanh x/2}{1 - \tanh^2 x/2}$$

$$\sinh x = \frac{2z}{1 - z^2}$$

Similarly

$$\cosh x = \frac{1 + z^2}{1 - z^2}$$

(137) 4.6

$$\tanh x = \frac{2z}{1+z^2}$$

14

$$\int \frac{1}{a + b \cosh x} dx$$

put $x = \tanh^{-1} z$

$$dx = \frac{2dz}{1-z^2}; \quad \cosh x = \frac{1+z^2}{1-z^2}$$

$$I = \int \frac{2dz/(1-z^2)}{a + b \left(\frac{1+z^2}{1-z^2} \right)}$$

$$= \int \frac{2dz}{a(1-z^2) + b(1+z^2)}$$

$$= \int \frac{2dz}{a - az^2 + b + bz^2}$$

$$= \int \frac{2dz}{(b-a)z^2 + (b+a)}$$

Case I if $a < b$

$\therefore b-a$ is true

$$\frac{2}{b-a} \int \frac{dz}{z^2 + \left(\sqrt{b+a/b-a} \right)^2}$$

$$= \frac{2}{b-a} \cdot \frac{1}{\sqrt{\frac{b+a}{b-a}}} \tan^{-1} \left(\frac{z}{\sqrt{\frac{b+a}{b-a}}} \right) + C$$

$$\frac{2}{\sqrt{b^2-a^2}} \tan^{-1} \left(\frac{\sqrt{b-a} z}{\sqrt{b+a}} \right) + C$$

$$\frac{2}{\sqrt{b^2-a^2}} \tan^{-1} \left(\frac{\sqrt{b-a} \tanh(x/2)}{\sqrt{b+a}} \right) + C$$

Case II:- if $a > b$

$\therefore a - b$ is true.

$$\int \frac{2dz}{-(a-b)z^2 + b + a} \Rightarrow \int \frac{2dz}{(b+a) - (a-b)z^2}$$

$$\frac{1}{a-b} \int \frac{2dz}{\left(\sqrt{\frac{a+b}{a-b}}\right)^2 - (z)^2}$$

$$\therefore \int \frac{1}{a^2 - z^2} dz = \frac{1}{2a} \ln \left| \frac{z+a}{z-a} \right|$$

$$= \frac{2}{a-b} \frac{1}{2\sqrt{\frac{a+b}{a-b}}} \ln \left| \frac{z + \sqrt{a+b/a-b}}{z - \sqrt{a+b/a-b}} \right| + c$$

$$\frac{1}{a-b} \frac{1}{\sqrt{a+b/a-b}} \ln \left| \frac{\sqrt{a-b}z + \sqrt{a+b}}{\sqrt{a-b}z - \sqrt{a+b}} \right|$$

$$= \frac{1}{\sqrt{a^2 - b^2}} \ln \left| \frac{\sqrt{a+b} + \sqrt{a-b} \tan x/2}{\sqrt{a+b} - \sqrt{a-b} \tan x/2} \right| + c$$

15

$$\int \frac{\cot x}{1 + \sin x} dx$$

$$\int \frac{\cos x}{\sin x (1 + \sin x)} dx$$

put $\sin x = t$

$$\cos x dx = dt$$

$$\int \frac{dt}{t(1+t)}$$

Take

$$\frac{1}{t(1+t)} = \frac{A}{t} + \frac{B}{1+t}$$

$$1 = A(1+t) + B(t)$$

(139) 4.6

putting $t = 0$

$$1 = A(1 + 0)$$

$$A = 1$$

putting $1 + t = 0 \Rightarrow t = -1$

$$1 = B(-1)$$

$$B = -1$$

$$\int \frac{dt}{t(1+t)} = \int \frac{1}{t} dt - \int \frac{1}{1+t} dt$$

$$= \ln|t| - \ln|1+t|$$

$$= \ln \left| \frac{t}{1+t} \right|$$

$$= \ln \left| \frac{\sin x}{1 + \sin x} \right| \Rightarrow \ln \left| \frac{2 \sin x/2 \cos x/2}{1 + 2 \sin x/2 \cos x/2} \right|$$

$$\ln \left| \frac{2 \sin x/2 \cos x/2}{\sin^2 x/2 + \cos^2 x/2 + 2 \sin x/2 \cos x/2} \right|$$

$$\ln \frac{2 \sin x/2 \cos x/2}{(\cos x/2 + \sin x/2)^2}$$

Dividing $\cos^2 x/2$

$$\ln \frac{2 \tan x/2}{(1 + \tan x/2)^2}$$

$$= \ln|2 \tan x/2| - 2 \ln|1 + \tan x/2| + c$$

16

$$\int \frac{2 - \cos x}{2 + \cos x} dx$$

$$= \left(-1 + \frac{4}{2 + \cos x} \right)$$

$$\begin{array}{r} \cos x + 2 \quad -1 \\ \hline -\cos x + 2 \\ + \cos x + 2 \\ \hline 4 \end{array}$$

Now by integration

$$-\int 1 dx + \int \frac{4}{2 + \cos x} dx$$

$$= -x + \int \frac{4}{2 + \cos x} dx \quad (i)$$

Now let

$$I_1 = \int \frac{4}{2 + \cos x} dx$$

$$\text{put } z = \tan x/2$$

$$dx = \frac{2 dz}{1 + z^2}$$

$$\cos x = \frac{1 - z^2}{1 + z^2}$$

$$= \int \frac{4 (2 dz / (1 + z^2))}{2 + (1 - z^2 / (1 + z^2))}$$

$$8 \int \frac{dz}{2 + 2z^2 + 1 - z^2}$$

$$= 8 \int \frac{dz}{z^2 + 3}$$

$$8 \int \frac{dz}{(z)^2 + (\sqrt{3})^2}$$

$$\frac{8}{\sqrt{3}} \tan^{-1} \left(\frac{z}{\sqrt{3}} \right)$$

$$\int \frac{4}{2 + \cos x} dx = \frac{8}{\sqrt{3}} \tan^{-1} \left(\frac{\tan x/2}{\sqrt{3}} \right)$$

putting this value in equation (i)

$$= -x + \frac{8}{\sqrt{3}} \tan^{-1} \left(\frac{\tan x/2}{\sqrt{3}} \right) + C$$

17

$$z = \tan(x/2)$$

$$\sin x = \frac{2z}{1+z^2}$$

$$\cos x = \frac{1-z^2}{1+z^2}$$

$$= \int \frac{2dz/1+z^2}{1 + \frac{2z}{1+z^2} + \frac{1-z^2}{1+z^2}} \Rightarrow \int \frac{2dz/1+z^2}{\frac{1+z^2+2z+1-z^2}{1+z^2}}$$

$$= \int \frac{2dz}{2z+2} \Rightarrow \int \frac{dz}{z+1}$$

$$= \ln |z+1| + C$$

$$\ln |\tan x/2 + 1| + C$$

18

$$\int \frac{\cos x}{2 - \cos x} dx$$

$$-\int 1 dx + \int \frac{2}{2 - \cos x} dx \quad \begin{array}{l} -\cos x + 2 \\ \hline \cos x \\ + \cos x + 2 \end{array}$$

$$-x + \int \frac{2}{2 - \cos x} dx \quad \text{--- (i)} \quad 2$$

Take

$$\int \frac{2}{2 - \cos x} dx$$

$$\text{put } z = \tan \frac{x}{2}$$

$$dx = \frac{2dz}{1+z^2}$$

$$\cos x = \frac{1-z^2}{1+z^2}$$

$$= \int \frac{2 \cdot 2dz/1+z^2}{2 - (1-z^2/1+z^2)}$$

$$= \int \frac{4dz/1+z^2}{2+2z^2-1+z^2} \Rightarrow \int \frac{4dz}{3z^2+1}$$

$$\int \frac{4dz}{3z^2+1} \Rightarrow \frac{4}{3} \int \frac{dz}{z^2 + 1/3}$$

$$\frac{4}{3} \int \frac{dz}{z^2 + (1/\sqrt{3})^2} \Rightarrow \frac{4}{3} \cdot \frac{1}{1/\sqrt{3}} \tan^{-1} \left(\frac{z}{1/\sqrt{3}} \right)$$

$$\frac{4}{3} \sqrt{3} \tan^{-1} \left(\frac{\sqrt{3} \tan x/2}{1} \right)$$

Now use this value in (i)

$$-x + \frac{4}{3} \sqrt{3} \tan^{-1}(\sqrt{3} \tan x/2) + C$$

Do

$$\int \frac{\cos x \, dx}{2 - \cos x} = -x + \frac{4}{\sqrt{3}} \tan^{-1}(\sqrt{3} \tan x/2) + C$$

9

$$\int \frac{1}{4\sin x - 3\cos x} dx$$

$$\frac{dx}{1+z^2} = \frac{2dz}{1+z^2}, \quad \sin x = \frac{2z}{1+z^2}$$

$$\cos x = \frac{1-z^2}{1+z^2}$$

$$\int \frac{2dz/1+z^2}{4(2z/1+z^2) - 3(1-z^2/1+z^2)}$$

(143) 4.6

$$\int \frac{2dz/1+z^2}{8z-3+3z^2/1+z^2} \Rightarrow \int \frac{2dz}{3z^2+8z-3}$$

$$\frac{2}{3} \int \frac{dz}{z^2 + (8/3)z - 1} \Rightarrow \frac{2}{3} \int \frac{dz}{z^2 + (8/3)z + (4/3)^2 - (4/3)^2 - 1}$$

$$\frac{2}{3} \int \frac{dz}{(z + 4/3)^2 - \frac{16}{9} - 1} \Rightarrow \frac{2}{3} \int \frac{dz}{(z + 4/3)^2 - 25/9}$$

$$\frac{2}{3} \int \frac{dz}{(z + 4/3)^2 - (5/3)^2} \Rightarrow \frac{2}{3} \cdot \frac{3}{2 \times 5} \ln \left| \frac{(z + 4/3) - (5/3)}{(z + 4/3) + (5/3)} \right|$$

$$\frac{1}{5} \ln \left| \frac{\tan x/2 - 1/3}{\tan x/2 + 4/3} \right| \Rightarrow \frac{1}{5} \ln \left| \frac{3 \tan x/2 - 1}{3 \tan x/2 + 4} \right|$$

$$\text{Ans. } \frac{1}{5} \ln \left| \frac{3 \tan x/2 - 1}{3 \tan x/2 + 4} \right|$$

20

$$\int \frac{dx}{\tan x - \sin x}$$

$$z = \tan x/2, \quad \sin x = \frac{2z}{1+z^2}$$

$$\cos x = \frac{1-z^2}{1+z^2}, \quad dx = \frac{2dz}{1+z^2}$$

$$\tan x = \frac{2z}{1-z^2}$$

$$= \int \frac{2dz/1+z^2}{\frac{2z}{1-z^2} - \frac{2z}{1+z^2}} \Rightarrow \int \frac{2dz/1+z^2}{2z \left(\frac{1}{1-z^2} - \frac{1}{1+z^2} \right)}$$

$$= \int \frac{2 dz / (1+z^2)}{2z \left[\frac{1+z^2-1+z^2}{(1-z^2)(1+z^2)} \right]} \Rightarrow \int \frac{2 dz / (1+z^2)}{2z \left(\frac{2z^2}{(1-z^2)(1+z^2)} \right)}$$

$$\int \frac{2 dz / (1+z^2)}{4z^2 \frac{1}{(1+z^2)(1-z^2)}} \Rightarrow \int \frac{dz}{2z^3 / (1-z^2)}$$

$$\frac{1}{2} \int \left(\frac{1-z^2}{z^3} \right) dz \Rightarrow \frac{1}{2} \int \left(\frac{1}{z^3} - \frac{z^2}{z^3} \right) dz$$

$$= \frac{1}{2} \int z^{-3} dz - \frac{1}{2} \int \frac{1}{z} dz$$

$$= \frac{1}{2} \frac{z^{-3+1}}{-3+1} - \frac{1}{2} \ln|z| + c$$

$$= \frac{1}{2} \frac{z^{-2}}{-2} - \frac{1}{2} \ln|z| + c$$

$$= -\frac{1}{4} z^{-2} - \frac{1}{2} \ln|z| + c$$

$$= -\frac{1}{4} \left[\tan\left(\frac{x}{2}\right) \right]^{-2} - \frac{1}{2} \ln \left| \tan\left(\frac{x}{2}\right) \right| + c$$

So

$$\int \frac{dx}{\tan x - \sin x} = -\frac{1}{4} \left(\tan \frac{x}{2} \right)^{-2} - \frac{1}{2} \ln \left(\tan \frac{x}{2} \right) + c$$

2.1

$$\int \frac{dx}{2 \cosh x + \sinh x}$$

$$\int \frac{dx}{2(\cosh^2 x/2 + \sinh^2 x/2) + 2 \sinh x/2 \cosh x/2}$$

Multiplying and dividing by $\cosh^2 x/2$

$$\frac{1}{2} \int \frac{\cosh^2 x/2 dx}{(1 + \tanh^2 x/2) + \tanh x/2}$$

2.2

$$\frac{1}{2} \int \frac{dx}{\cosh^2 x/2 (1 + \tanh^2 x/2) + \tanh x/2}$$

$$\frac{1}{2} \int \frac{\operatorname{sech}^2 x/2}{(1 + \tanh^2 x/2) + \tanh x/2}$$

$$\int \frac{1/2 \operatorname{sech}^2(x/2)}{(1 + \tanh^2 x/2) + \tanh x/2}$$

$$\tanh x = u$$

$$\frac{\operatorname{sech}^2 x}{2} \cdot \frac{1}{2} dx = du$$

$$\int \frac{du}{u + (1 + u^2)} \Rightarrow \int \frac{du}{u^2 + u + 1}$$

$$\int \frac{du}{u^2 + 2(u)(1/2) + (1/2)^2 - (1/2)^2 + 1}$$

$$\int \frac{du}{(u + 1/2)^2 + (\sqrt{3}/2)^2}$$

$$\frac{1}{\sqrt{3}/2} \tan^{-1} \left(\frac{u + 1/2}{\sqrt{3}/2} \right) \Rightarrow \frac{2}{\sqrt{3}} \tan^{-1} \left[\frac{2u + 1/2}{\sqrt{3}/2} \right]$$

$$\frac{2}{\sqrt{3}} \tan^{-1} \left[\frac{2u + 1}{\sqrt{3}} \right]$$

Ans

$$\int \frac{dx}{2 \cosh x + \sinh x} = \frac{2}{\sqrt{3}} \tan^{-1} \left[\frac{2 \tanh(x/2) + 1}{\sqrt{3}} \right] + C$$

22

$$\int \frac{\sin x + \cos x}{\tan x} dx$$

Sol:- $\int \left(\frac{\sin x}{\tan x} + \frac{\cos x}{\tan x} \right) dx$

$$\int \left(\frac{\sin x}{\sin x / \cos x} + \frac{\cos x}{\sin x / \cos x} \right) dx$$

$$\int \left(\cos x + \frac{\cos^2 x}{\sin x} \right) dx$$

$$\int \cos x dx + \int \frac{(1 - \sin^2 x)}{\sin x} dx$$

$$\sin x + \int \left(\frac{1}{\sin x} - \frac{\sin^2 x}{\sin x} \right) dx$$

$$\sin x + \int \operatorname{cosec} x dx - \int \sin x dx$$

$$\sin x + \ln |\operatorname{cosec} x - \cot x| + \cos x + C$$

$$\sin x + \cos x + \ln \left| \frac{1}{\sin x} - \frac{\cos x}{\sin x} \right| + C$$

$$\sin x + \cos x + \ln \left| \frac{1 - \cos x}{\sin x} \right| + C$$

$$\sin x + \cos x + \ln \left| \frac{\sin^2 x/2 + \cos^2 x/2 - (\cos^2 x/2 - \sin^2 x/2)}{2 \sin x/2 \cos x/2} \right|$$

$$\sin x + \cos x + \ln \left| \frac{\sin^2 x/2 + \cos^2 x/2 - \cos^2 x/2 + \sin^2 x/2}{2 \sin x/2 \cos x/2} \right|$$

$$\sin x + \cos x + \ln \left| \frac{\sin x/2}{\cos x/2} \right| + C$$

$$\sin x + \cos x + \ln |\tan x/2| + C$$

2.0

$$\int \frac{\sin x + \cos x}{\tan x} dx = \sin x + \cos x + \ln |\tan x/2| + C$$

2.4

2.3

$$\int \cos x \sqrt{1 - \cos x} dx$$

$$= \int (1 - 2 \sin^2 x/2) \sqrt{2 \sin^2 x/2} dx$$

$$\int \sqrt{2} \sin x/2 (1 - 2 \sin^2 x/2) dx$$

$$\sqrt{2} \int \sin x/2 dx - 2\sqrt{2} \int \sin^2 x/2 \sin x/2 dx$$

$$\sqrt{2} \left(\frac{-\cos x/2}{1/2} \right) - 2\sqrt{2} \int (1 - \cos^2 x/2) \sin x/2 dx$$

$$-2\sqrt{2} \cos x/2 - 2\sqrt{2} \int \sin x/2 dx + 2\sqrt{2} \int \cos^2 x/2 \sin x/2 dx$$

$$-2\sqrt{2} \cos x/2 + 4\sqrt{2} \cos x/2 - 4\sqrt{2} \int \cos^2 x/2 (-1/2 \sin x/2) dx$$

$$-2\sqrt{2} \cos x/2 + 4\sqrt{2} \cos x/2 - 4\sqrt{2} \frac{\cos^3 x/2}{3}$$

$$2\sqrt{2} \cos x/2 - \frac{4\sqrt{2} \cos^3 x/2}{3} + C$$

$$2\sqrt{2} \cos x/2 \left[1 - \frac{2}{3} \cos^2 x/2 \right]$$

$$(2) \quad \frac{2\sqrt{2} \cos x/2 (3 - 2 \cos^2 x/2)}{3} \Rightarrow \frac{2\sqrt{2}}{3} \sqrt{\frac{1 + \cos x}{2}} [3 - (1 + \cos x)]$$

$$\frac{2\sqrt{2}}{3} \frac{\sqrt{1 + \cos x}}{\sqrt{2}} [3 - 1 - \cos x] \Rightarrow \frac{2}{3} \sqrt{1 + \cos x} (2 - \cos x) + C$$

DO

$$\int \cos x \sqrt{1 - \cos x} dx = \frac{2}{3} (2 - \cos x) \sqrt{1 + \cos x} + C$$

$$\int \sqrt{a + \sec^2 x} dx$$

$$\therefore dx = \frac{2 dz}{\sec^2 x \tan x}$$

$$\text{put } \sqrt{a + \sec^2 x} = z$$

by squaring

$$a + \sec^2 x = z^2$$

$$\Rightarrow \sec^2 x = z^2 - a$$

$$\sec x \cdot \sec x \tan x dx = 2z dz$$

$$dx = \frac{2 dz}{\sec^2 x \sqrt{\sec^2 x - 1}}$$

$$= \frac{z dz}{(z^2 - a) \sqrt{z^2 - a - 1}}$$

$$= \int \frac{z dz \cdot z}{(z^2 - a) \sqrt{z^2 - (a+1)}}$$

$$\therefore dx = \frac{z dz}{(z^2 - a) \sqrt{z^2 - (a+1)}}$$

$$\int \frac{z^2 dx}{(z^2 - a) \sqrt{z^2 - (a+1)}} \Rightarrow \int \frac{z^2}{z^2 - a} \cdot \frac{1}{\sqrt{z^2 - (a+1)}}$$

$$\int \left(\frac{1+a}{z^2 - a} \right) \frac{dz}{\sqrt{z^2 - (a+1)}}$$

$$= \int \frac{dz}{\sqrt{z^2 - (a+1)}} + a \int \frac{dz}{(z^2 - a) \sqrt{z^2 - (a+1)}}$$

$$I = I_1 + a I_2$$

Take

$$I_1 = \int \frac{dz}{\sqrt{z^2 - (\sqrt{a+1})^2}}$$

$$I_1 = \ln \left| \frac{z + \sqrt{z^2 - (a+1)}}{\sqrt{a+1}} \right| \Rightarrow \ln \left| \frac{\sqrt{a + \sec^2 x} + \tan x}{\sqrt{a+1}} \right|$$

New

$$I_2 = \int \frac{dz}{(z^2 - a) \sqrt{z^2 - (a+1)}}$$

$$z = \frac{1}{t} \Rightarrow dz = \frac{-1}{t^2} dt$$

$$z^2 - a = \frac{1}{t^2} - a \Rightarrow \frac{1 - at^2}{t^2}$$

$$z^2 - (a+1) = \frac{1}{t^2} - (a+1) \Rightarrow \frac{1 - (a+1)t^2}{t^2}$$

$$I_2 = \frac{-1/t^2 dt}{\frac{1 - at^2}{t^2} \cdot \frac{\sqrt{1 - (a+1)t^2}}{t}}$$

$$= \frac{-t dt}{(1 - at^2) \sqrt{1 - (a+1)t^2}}$$

$$\Rightarrow \sqrt{1 - (a+1)t^2} = u$$

$$1 - \cos \theta = 2 \sin^2 \theta/2$$

$$\begin{aligned} \cos \alpha - \cos \theta &= (2 \cos^2 \alpha/2 - 1) - (2 \cos^2 \theta/2 - 1) \\ &= 2 \cos^2 \alpha/2 - 1 - 2 \cos^2 \theta/2 + 1 \\ &= 2 \cos^2 \alpha/2 - 2 \cos^2 \theta/2 \end{aligned}$$

$$I = \int \frac{2 \sin^2 \theta/2}{\sqrt{2 \cos^2 \alpha/2 - 2 \cos^2 \theta/2}} d\theta$$

$$I = \int \frac{\sin \theta/2}{\sqrt{\cos^2 \alpha/2 - \cos^2 \theta/2}} d\theta$$

26(i)

Taking common from denominator $\sqrt{\cos^2 \alpha/2 \cos^2 \theta/2}$

$$= \int \frac{\sin \theta/2}{\cos \alpha/2 \cos \theta/2 \sqrt{1/\cos^2 \theta/2 - 1/\cos^2 \alpha/2}}$$

$$\frac{1}{\cos \alpha/2} \int \frac{\tan \theta/2}{\sqrt{\sec^2 \theta/2 - \sec^2 \alpha/2}}$$

$$\sec \alpha/2 \int \frac{\tan \theta/2}{\sqrt{\sec^2 \theta/2 - \sec^2 \alpha/2}}$$

$$\sqrt{\sec^2 \theta/2 - \sec^2 \alpha/2} = u$$

$$\sec^2 \theta/2 - \sec^2 \alpha/2 = u^2$$

$$\sec^2 \theta/2 = u^2 + \sec^2 \alpha/2$$

$$2 \sec \theta/2 \cdot \sec \theta/2 \tan \theta/2 \cdot 1/2 d\theta = 2u du + 0$$

$$\tan \theta/2 d\theta = 2u du / \sec^2 \theta/2$$

$$\tan \theta/2 d\theta = 2u du / (u^2 + \sec^2 \alpha/2)$$

$$= \sec \alpha/2 \int \frac{2u du}{(u^2 + \sec^2 \alpha/2) u}$$

$$2 \sec \alpha/2 \int \frac{1}{u^2 + \sec^2 \alpha/2} du$$

(ii)

(149) 4.6

$$1 - (a+1)t^2 = u^2 \Rightarrow -2(a+1)t dt = 2u du$$

$$-t dt = u du / (a+1) \Rightarrow -t dt = u du$$

$$1 - at^2 = 1 - a \left[\frac{1-u^2}{a+1} \right] \Rightarrow \frac{a+1-a+au^2}{a+1}$$

$$1 - at^2 = \frac{1+au^2}{a+1}$$

$$\Rightarrow 1 - (a+1)t^2 = 1 - (a+1) \left(\frac{1-u^2}{a+1} \right) \Rightarrow 1 - 1 + u^2$$

$$1 - (a+1)t^2 = u^2$$

$$I_2 = \int \frac{u du / (a+1)}{\frac{1+au^2}{a+1} - \sqrt{u^2}} \Rightarrow \int \frac{du}{1+au^2}$$

$$\frac{1}{a} \int \frac{du}{\frac{1}{a} + u^2} \Rightarrow \frac{1}{a} \int \frac{du}{u^2 + (\frac{1}{\sqrt{a}})^2}$$

$$\frac{1}{a} \tan^{-1} \left(\frac{u}{\frac{1}{\sqrt{a}}} \right) \Rightarrow \frac{1}{a} \tan^{-1} (\sqrt{a} u)$$

$$\frac{1}{a} \tan^{-1} (\sqrt{a} \sqrt{1-(a+1)t^2}) \quad \because u^2 = 1-(a+1)t^2$$

$$\frac{1}{a} \tan^{-1} (\sqrt{a} \sqrt{1-(a+1) \cdot \frac{1}{z^2}}) \quad \because t = 1/z \quad \because z^2 = a + \sec^2 x$$

$$\frac{1}{a} \tan^{-1} \left(\frac{\sqrt{a} \sqrt{z^2 - (a+1)}}{z} \right) \Rightarrow \frac{1}{a} \tan^{-1} \left(\frac{\sqrt{a} \sqrt{\sec^2 x + a - a - 1}}{\sqrt{a + \sec^2 x}} \right)$$

$$I_2 = \frac{1}{a} \tan^{-1} \left(\frac{\sqrt{a} \tan x}{\sqrt{a + \sec^2 x}} \right)$$

put I_1 & I_2 in (i)

$$I = \ln \left| \frac{\sqrt{\sec^2 x + a} + \tan x}{\sqrt{a+1}} \right| + a \cdot \frac{1}{a} \tan^{-1} \left(\frac{\sqrt{a} \tan x}{\sqrt{a + \sec^2 x}} \right) + c$$

$$I = \ln \left| \frac{\sqrt{\sec^2 x + a} + \tan x}{\sqrt{a+1}} \right| + \tan^{-1} \left(\frac{\sqrt{a} \tan x}{\sqrt{a + \sec^2 x}} \right) + c \text{ Ans.}$$

2.5

$$I = \int \frac{1 - \cos \theta}{\cos \theta - \cos \theta} d\theta$$

(151) 4.6

$$= 2 \sec \alpha/2 \tan^{-1} \left(\frac{u}{\sec \alpha/2} \right)$$

$$= 2 \sec \alpha/2 \tan^{-1} \left(\frac{\sqrt{\sec^2 \theta/2 - \sec^2 \alpha/2}}{\sec \alpha/2} \right)$$

$$= 2 \sec \alpha/2 \tan^{-1} \left[\frac{\sec \theta/2 \sec \alpha/2 \sqrt{1/\sec^2 \alpha/2 - 1/\sec^2 \theta/2}}{\sec \alpha/2} \right]$$

$$= 2 \sec \alpha/2 \tan^{-1} \left[\frac{\sqrt{\cos^2 \alpha/2 - \cos^2 \theta/2}}{\cos \theta/2} \right] + C$$

2.6 (i)

$$z = \tan(x/2), \quad dx = 2dz/(1+z^2)$$

$$\cos x = 1 - z^2/(1+z^2) \Rightarrow \sec x = 1 + z^2/(1-z^2)$$

$$\int \sec x dx = \int \frac{(1+z^2) + 2dz}{(1-z^2)(1+z^2)}$$

$$= 2 \int \frac{dz}{1-z^2} \Rightarrow 2 \cdot \frac{1}{2} \ln \left| \frac{1+z}{1-z} \right|$$

$$\ln \left| \frac{1+\tan x/2}{1-\tan x/2} \right| \Rightarrow \ln \left| \frac{1 + \sin x/2 / \cos x/2}{1 - \sin x/2 / \cos x/2} \right|$$

$$= \ln \left| \frac{\cos x/2 + \sin x/2}{\cos x/2 - \sin x/2} \right| \Rightarrow \frac{1}{2} \ln \left| \frac{\cos x/2 + \sin x/2}{\cos x/2 - \sin x/2} \right|$$

$$\frac{1}{2} \ln \left| \frac{\cos^2 x/2 + \sin^2 x/2 + 2 \sin x/2 \cos x/2}{\cos^2 x/2 + \sin^2 x/2 - 2 \sin x/2 \cos x/2} \right|$$

$$\frac{1}{2} \ln \left| \frac{1 + \sin x}{1 - \sin x} \right|$$

$$\int \sec x dx = \frac{1}{2} \ln \left| \frac{1 + \sin x}{1 - \sin x} \right| \text{ Ans.}$$

(ii)

$$\int \operatorname{cosec} x dx$$

$$z = \tan x/2$$

$$dx = 2dz/(1+z^2)$$

$$\cos x = \frac{1-z^2}{1+z^2}$$

$$\sin x = \frac{2z}{1+z^2}$$

$$\operatorname{Cosec} x dx = \frac{1+z^2}{2z}$$

$$\operatorname{Cosec} x dx = \int \frac{1+z^2}{2z} \times \frac{2dz}{1+z^2}$$

$$= \int \frac{1}{z} dz \Rightarrow \ln|z| + c$$

$$\ln|\tan x/2| \Rightarrow \ln|\sin x/2 / \cos x/2|$$

$$\frac{1}{2} \cdot 2 \ln \left| \frac{\sin x/2}{\cos x/2} \right| \Rightarrow \frac{1}{2} \ln \left| \frac{\sin x/2}{\cos x/2} \right|^2$$

$$\frac{1}{2} \ln \left| \frac{\sin^2 x/2}{\cos^2 x/2} \right| \Rightarrow \frac{1}{2} \ln \left| \frac{1 - \cos x/2}{1 + \cos x/2} \right|$$

$$\int \operatorname{cosec} x dx = \frac{1}{2} \ln \left| \frac{1 - \cos x}{1 + \cos x} \right| \text{ Ans.}$$

$$\int (\sqrt{\tan x} + \sqrt{\cot x}) dx$$

$$\int (\sqrt{\tan x} + 1/\sqrt{\tan x}) dx$$

$$\int \left(\frac{\tan x + 1}{\sqrt{\tan x}} \right) dx$$

$$\Rightarrow \sqrt{\tan x} = t \Rightarrow \tan x = t^2$$

$$\sec^2 x dx = 2t dt \Rightarrow dx = \frac{2t dt}{\sec^2 x}$$

$$dx = \frac{2t dt}{1 + \tan^2 x}$$

$$dx = \frac{2t}{1 + (t^2)^2} dt \Rightarrow dx = \frac{2t}{1 + t^4} dt$$

$$\int \frac{t^2 + 1}{t} \cdot \frac{2t}{(1 + t^4)} dt \Rightarrow 2 \int \frac{t^2 + 1}{1 + t^4} dt$$

$$2 \int \frac{t^2 (1 + 1/t^2)}{t^2 (t^2 + 1/t^2)} dt$$

$$2 \int \frac{(1 + 1/t^2)}{t^2 + 1/t^2} dt$$

$$t - \frac{1}{t} = u \Rightarrow (1 + \frac{1}{t^2}) dt = du$$

by squaring

$$\frac{t^2+1}{t^2} - 2 = u^2 \Rightarrow \frac{t^2+1}{t^2} = u^2 + 2$$

$$= 2 \int \frac{du}{u^2 + 2} \Rightarrow 2 \int \frac{1}{(u)^2 + (\sqrt{2})^2} du$$

$$2 \cdot \frac{1}{\sqrt{2}} \tan^{-1}(u/\sqrt{2}) \Rightarrow \sqrt{2} \tan^{-1}\left(\frac{t - 1/t}{\sqrt{2}}\right)$$

$$\sqrt{2} \tan^{-1}\left(\frac{\tan x - 1}{\sqrt{2} \sqrt{\tan x}}\right) \Rightarrow \sqrt{2} \tan^{-1}\left(\frac{\tan x - 1}{\sqrt{2} \sqrt{\tan x}}\right)$$

Do

$$= \sqrt{2} \tan^{-1}\left(\frac{\tan x - 1}{\sqrt{2} \sqrt{\tan x}}\right)$$

Now

(Method of Sir Farooq)

$$\int (\sqrt{\tan x} + \sqrt{\cot x}) dx$$

$$\int \left(\frac{\sqrt{\sin x}}{\sqrt{\cos x}} + \frac{\sqrt{\cos x}}{\sqrt{\sin x}} \right) dx$$

$$\int \frac{\sin x + \cos x}{\sqrt{\sin x \cos x}} dx$$

$$\text{let } \sin x - \cos x = t$$

$$(\cos x + \sin x) dx = dt$$

$$\sin^2 x + \cos^2 x - 2 \sin x \cos x = t^2$$

$$1 - 2 \sin x \cos x = t^2$$

$$\frac{1 - t^2}{2} = \sin x \cos x$$

$$\sqrt{\frac{1 - t^2}{2}} = \sqrt{\sin x \cos x}$$

$$= \int \frac{dt}{\sqrt{1-t^2}}$$

(154) 4-6

$$\sqrt{2} \int \frac{1}{\sqrt{1-t^2}}$$

$$\sqrt{2} \sin^{-1}(t)$$

$$\sqrt{2} \sin^{-1}(\sin x - \cos x) + C \text{ Ans.}$$

~~1~~

~~pc~~

v
v