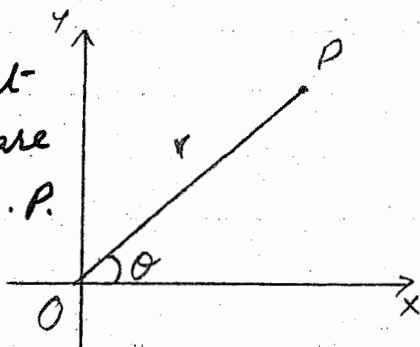


Exercise 6.3

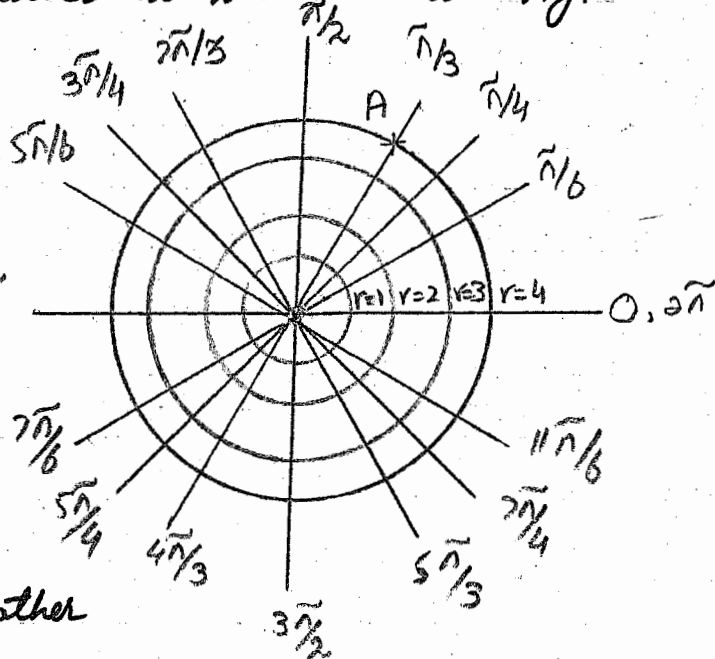
Polar Coordinates.

If P is a point, such that $|OP| = r$ and $\hat{POX} = \theta$ then (r, θ) are called Polar coordinates of the pt. P . The curves in (r, θ) are called the Polar curves.



To draw a graph of polar curves, we select a pt. O of the plane, which is called the pole then we draw the family of curves as shown in the Fig. and also a family of straight lines through the pole as in the Fig.

Now if a pt. A has $\tilde{r}$ the Polar coordinates, $(4, \tilde{r}_3)$, then the position of this pt. A is shown in the Fig. Similarly all other pts. can be drawn.



Relation between Polar and Cartesian Coordinates:-

Consider the pt. P whose cartesian coordinates are (x, y) and polar coordinates are (r, θ) , then

$|OA| = x$; $|AP| = y$; $|OP| = r$ and

$$\hat{AOP} = \theta$$

Then $\sin \theta = \frac{y}{r}$ and $\cos \theta = \frac{x}{r}$

$$\Rightarrow x = r \cos \theta \text{ and } y = r \sin \theta$$

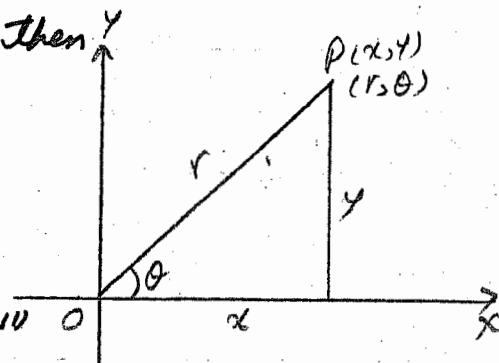
$$(i)^2 + (ii)^2 \Rightarrow \text{--- (i)}$$

$$x^2 + y^2 = r^2 \cos^2 \theta + r^2 \sin^2 \theta$$

$$\Rightarrow r = \sqrt{x^2 + y^2} \text{ --- (iii)}$$

$$(ii) \div (i) \Rightarrow$$

$$\frac{y}{x} = \frac{r \sin \theta}{r \cos \theta} = \tan \theta$$



$$\Rightarrow \theta = \tan^{-1} \frac{y}{x} \quad \text{--- (iv)}$$

(i), (ii) gives cartesian in form of polar and (iii), (iv) polar in form of cartesian.

Note:-

$$\sin \theta = \frac{y}{\sqrt{x^2 + y^2}} = \frac{y}{r}$$

$$\cos \theta = \frac{x}{\sqrt{x^2 + y^2}} = \frac{x}{r}$$

Exercise 6.3.

In Problems 1-8, express the given equations in rectangular coordinates.

Question 1.

Soln.

$$r^2 = a^2 \sin 2\theta \quad \text{--- (1)}$$

$$r^2 = 2a^2 \sin \theta \cos \theta \quad \text{--- (2)}$$

$$\therefore r^2 = x^2 + y^2, \quad \sin \theta = \frac{y}{r} \text{ and } \cos \theta = \frac{x}{r}$$

$\therefore$ (2) becomes

$$r^2 = 2a^2 \frac{y}{r} \cdot \frac{x}{r}$$

$$r^4 = 2a^2 xy$$

$$(x^2 + y^2)^2 = 2a^2 xy$$

Question 2.

Soln.

$$r^4 \sin 4\theta = a^4$$

$$r^4 = \frac{a^4}{\sin 4\theta}$$

$$r^4 = \frac{a^4}{2 \sin 2\theta \cos 2\theta}$$

$$r^4 = \frac{a^4}{4 \sin \theta \cos \theta (\cos^2 \theta - \sin^2 \theta)} \quad \text{--- (1)}$$

$$\therefore \sin \theta = \frac{y}{r} \text{ and } \cos \theta = \frac{x}{r}$$

$$\therefore (1) \Rightarrow = \frac{a^4}{4 \left(\frac{y}{r}\right) \left(\frac{x}{r}\right) \left(\frac{x^2}{r^2} - \frac{y^2}{r^2}\right)}$$

$$= \frac{a^4}{\frac{4xy}{r^2} \left(\frac{x^2 - y^2}{r^2}\right)}$$

$$r^4 = \frac{a^4 r^4}{4xy(x^2 - y^2)}$$

$$\Rightarrow 4xy(x^2 - y^2) = a^4$$

Question 3.

$$r^2 = a^2 \cos 2\theta$$

Soln.

$$r^2 = a^2 (\cos^2 \theta - \sin^2 \theta)$$

$$r^2 = a^2 \left(\frac{x^2}{r^2} - \frac{y^2}{r^2} \right)$$

$$r^2 = a^2 \left(\frac{r^2 - y^2}{r^2} \right)$$

$$r^4 = a^2 (x^2 - y^2)$$

$$(x^2 + y^2)^2 = a^2 (x^2 - y^2)$$

$$\therefore \frac{x}{r} = \cos \theta, \frac{y}{r} = \sin \theta$$

$$\therefore r^2 = x^2 + y^2$$

Question 4.

$$r = 2a \sin \theta \sec \theta$$

Soln.

$$r = 2a \frac{y}{r} \cdot \frac{y}{x}$$

$$\therefore \sin \theta = \frac{y}{r}, \sec \theta = \frac{r}{x}$$

$$r^2 = \frac{2a y^2}{x}$$

$$x(x^2 + y^2) = 2a y^2$$

Question 5.

$$r = 1 - \cos \theta$$

Soln.

$$r = 1 - \frac{x}{r}$$

$$\therefore \cos \theta = \frac{x}{r}$$

$$r = \frac{r - x}{r}$$

$$r^2 = r - x$$

$\Rightarrow$

$$x^2 + y^2 = \sqrt{x^2 + y^2} - x$$

$$\therefore r^2 = x^2 + y^2$$

$$x^2 + y^2 + x = \sqrt{x^2 + y^2}$$

$$\Rightarrow r = \sqrt{x^2 + y^2}$$

$$(x^2 + y^2 + x)^2 = x^2 + y^2$$

Question 6.

$$r^2 (4 \sin^2 \theta - 9 \cos^2 \theta) = 36$$

Soln.

$$r^2 \left(4 \frac{y^2}{r^2} - 9 \frac{x^2}{r^2} \right) = 36$$

$$r^2 (4y^2 - 9x^2) = 36$$

$$4y^2 - 9x^2 = 36$$

$$\frac{4y^2}{36} - \frac{9x^2}{36} = 1$$

$$\Rightarrow \frac{y^2}{9} - \frac{x^2}{4} = 1$$

Question 7.

Soln.

$$r = \frac{8}{2 - \cos \theta}$$

$$r = \frac{8}{2 - \frac{x}{r}} = \frac{8}{\frac{2r - x}{r}}$$

$$r = \frac{8r}{2r - x} \Rightarrow 1 = \frac{8}{2r - x}$$

$$\Rightarrow 2r - x = 8$$

$$\Rightarrow 2r = x + 8$$

$$2\sqrt{x^2+y^2} = x + 8$$

$$(2)^2(x^2+y^2) = x^2 + 64 + 16x$$

$$4x^2 + 4y^2 = x^2 + 64 + 16x$$

$$\Rightarrow 3x^2 + 4y^2 - 16x - 64 = 0$$

$$\therefore r = \sqrt{x^2+y^2}$$

Question 8.

$$r = 2\sin\theta + 3\cos\theta$$

Soln.

$$r = 2\frac{y}{r} + 3\frac{x}{r}$$

$$\therefore \sin\theta = \frac{y}{r}$$

$$\cos\theta = \frac{x}{r}$$

$$\Rightarrow r^2 = 2y + 3x$$

$$\Rightarrow x^2 + y^2 = 2y + 3x$$

$$\therefore r^2 = x^2 + y^2$$

Transform the given equations in polar coordinates. (9-15)

Question 9.

$$xy = a \quad \text{--- (1)}$$

Soln.

$$\therefore x = r\cos\theta \text{ and } y = r\sin\theta$$

$$\therefore (1) \Rightarrow r\cos\theta \cdot r\sin\theta = a$$

$$r^2 \sin\theta \cos\theta = a$$

$$\Rightarrow r^2 = \frac{a}{\sin\theta \cos\theta}$$

$$\Rightarrow r^2 = a \operatorname{cosec}\theta \sec\theta$$

Question 10

$$y^2 = 4x$$

Soln.

$$r^2 \sin^2\theta = 4r\cos\theta$$

$$r(r\sin^2\theta - 4\cos\theta) = 0$$

$$r\sin^2\theta - 4\cos\theta = 0$$

$$\therefore y = r\sin\theta$$

$$x = r\cos\theta$$

Question 11

$$y = \frac{x}{x+1}$$

$$r\sin\theta = \frac{r\cos\theta}{r\cos\theta + 1}$$

$$r\sin\theta(r\cos\theta + 1) = r\cos\theta$$

$$\sin\theta(r\cos\theta + 1) = \cos\theta$$

$$r\sin\theta\cos\theta + \sin\theta = \cos\theta$$

$$r\sin\theta\cos\theta = \cos\theta - \sin\theta$$

$$r = \frac{\cos\theta - \sin\theta}{\sin\theta\cos\theta}$$

$$\Rightarrow r = \frac{1}{\sin \theta} - \frac{1}{\cos \theta}$$

$$r = \operatorname{cosec} \theta - \sec \theta.$$

Question 12.

$$x^2 + y^2 - 8x + 6y + 7 = 0 \quad \text{--- (1)}$$

Soln.

$$\therefore r^2 = x^2 + y^2, \quad x = r \cos \theta, \quad y = r \sin \theta$$

$$\therefore (1) \Rightarrow r^2 - 8r \cos \theta + 6r \sin \theta + 7 = 0$$

$$r^2 - 2r(4 \cos \theta + 3 \sin \theta) + 7 = 0$$

Question 13.

$$(x^2 + y^2) y^2 = a^2 x^2$$

Soln.

$$r^2 \cdot r^2 \sin^2 \theta = a^2 r^2 \cos^2 \theta$$

$$\therefore x^2 + y^2 = r^2$$

$$r^2 \sin^2 \theta = a^2 \cos^2 \theta$$

$$y = r \sin \theta$$

$$r^2 = \frac{a^2 \cos^2 \theta}{\sin^2 \theta}$$

$$x = r \cos \theta.$$

$$\Rightarrow r^2 = a^2 \cot^2 \theta$$

$$\Rightarrow r = a \cot \theta$$

Question 14.

$$x^3 + 4x^2 + xy^2 - 4y^3 = 0$$

Soln.

$$r^3 \cos^3 \theta + 4r^2 \cos^2 \theta + r \cos \theta \cdot r^2 \sin^2 \theta - 4r^3 \sin^3 \theta = 0$$

$$r^2 (r \cos^3 \theta + r \cos \theta \sin^2 \theta + 4 \cos^2 \theta - 4 \sin^3 \theta) = 0$$

$$r \cos^3 \theta + 4(\cos^2 \theta - \sin^2 \theta) + r \sin^2 \theta \cos \theta = 0$$

$$r \cos^3 \theta + 4 \cos^2 \theta + r \sin^2 \theta \cos \theta = 0$$

Question 15.

$$x^4 + 2x^2y^2 + y^4 - 6x^2y + 2y^3 = 0$$

Soln.

$$(x^2 + y^2)^2 - 2y(3x^2 + y^2) = 0$$

$$r^4 - 2r \sin \theta (3r^2 \cos^2 \theta + r^2 \sin^2 \theta) = 0$$

$$r^4 - 2r^3 \sin \theta (3 \cos^2 \theta + \sin^2 \theta) = 0$$

$$r - 2 \sin \theta (3 \cos^2 \theta + \sin^2 \theta) = 0$$