

Exercise 6.1

Examine whether each of the given equations represents two straight lines. If so, find an equation of each straight line.

Q.1

$$10xy + 8x - 15y - 12 = 0 \quad \text{--- ①}$$

General equation of 2nd degree is

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0 \quad \text{--- ②}$$

From ① and ②

$$a=0, h=5, b=0, g=4, f=-\frac{15}{2}, c=-12$$

Now

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = \begin{vmatrix} 0 & 5 & 4 \\ 5 & 0 & -\frac{15}{2} \\ 4 & -\frac{15}{2} & -12 \end{vmatrix}$$

$$= 0 - 5(-60 - (-30)) + 4(-\frac{75}{2} - 0)$$

$$= -5(-60 + 30) - 2(75)$$

$$= -5(-30) - 150$$

$$= 150 - 150$$

$$= 0$$

i.e ① represents two straight lines.

1) $\Rightarrow$

$$2x(5y+4) - 3(5y+4) = 0$$

$$(2x-3)(5y+4) = 0$$

$$2x-3=0, \quad 5y+4=0$$

are the two straight lines.

② # 2

$$2x^2 - xy + 5x - 2y + 2 = 0 \quad \text{--- ①}$$

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0 \quad \text{--- ②}$$

Here

$$a=2, h=-\frac{1}{2}, b=0, g=\frac{5}{2}, f=-1, c=2$$

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = \begin{vmatrix} 2 & -\frac{1}{2} & \frac{5}{2} \\ -\frac{1}{2} & 0 & -1 \\ \frac{5}{2} & -1 & 2 \end{vmatrix}$$

$$= 2(0-1) + \frac{1}{2}(-1 + \frac{5}{2}) + \frac{5}{2}(\frac{1}{2} - 0)$$

$$= -2 + \frac{1}{2} \left(\frac{-2+5}{2} \right) + \frac{5}{4}$$

$$= -2 + \frac{3}{4} + \frac{5}{4} = \frac{-8+3+5}{4} = 0$$

Hence the given equation represents two straight lines.

1) $\Rightarrow$

$$2x^2 + x(5-y) + 2(1-y) = 0$$

$$x = \frac{-(5-y) \pm \sqrt{(5-y)^2 - 4(2)(2-2y)}}{2(2)}$$

$$x = \frac{-(5-y) \pm \sqrt{(5-y)^2 - 8(2-2y)}}{4}$$

$$x = \frac{-(5-y) \pm \sqrt{25+y^2-10y-16+16y}}{4}$$

$$x = \frac{-(5-y) \pm \sqrt{9+y^2+6y}}{4}$$

$$x = \frac{-(5-y) \pm \sqrt{(3+y)^2}}{4}$$

$$x = \frac{-(5-y) \pm (3+y)}{4}$$

$$x = \frac{-5+y+3+y}{4} \text{ and } \frac{-5+y-3-y}{4}$$

$$x = \frac{-2+2y}{4} \text{ and } \frac{-8}{4}$$

$$x = \frac{y-1}{2} \text{ and } -\frac{8}{4}$$

$$\Rightarrow x = \frac{y-1}{2} \text{ and } x = -\frac{8}{4}$$

$$2x - y + 1 = 0 \text{ and } 4x + 8 = 0$$

are the two straight lines.

② #3

$$6x^2 - 17xy - 3y^2 + 22x + 10y - 8 = 0 \quad \text{--- ①}$$

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

Here $a = 6, h = -\frac{17}{2}, b = -3, g = 11, f = 5, c = -8$

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = \begin{vmatrix} 6 & -\frac{17}{2} & 11 \\ -\frac{17}{2} & -3 & 5 \\ 11 & 5 & -8 \end{vmatrix}$$

$$= 6(24 - 25) + \frac{17}{2}(68 - 55) + 11(-\frac{85}{2} + 33)$$

$$= -6 + \frac{221}{2} - \frac{935}{2} + 363$$

$$= \frac{221}{2} - \frac{935}{2} + 357$$

$$= \frac{221 - 935 + 714}{2} = \frac{0}{2} = 0$$

Hence the given equation represents the two straight lines.

1) $\Rightarrow 6x^2 + x(22 - 17y) + (-3y^2 + 10y - 8) = 0$

$$x = \frac{-(22 - 17y) \pm \sqrt{(22 - 17y)^2 - 4(6)(-3y^2 + 10y - 8)}}{2(6)}$$

$$x = \frac{-22 + 17y \pm \sqrt{484 + 289y^2 - 748y + 72y^2 - 240y + 192}}{12}$$

$$x = \frac{-22 + 17y \pm \sqrt{676 + 361y^2 - 988y}}{12}$$

$$x = \frac{-22 + 17y \pm \sqrt{(26 - 19y)^2}}{12}$$

$$x = \frac{-22 + 17y \pm (26 - 19y)}{12}$$

$$\Rightarrow x = \frac{-22 + 17y + 26 - 19y}{12}, \quad x = \frac{-22 + 17y - 26 + 19y}{12}$$

$$x = \frac{4 - 2y}{12}, \quad x = \frac{-48 + 36y}{12}$$

$$x = \frac{2-y}{6}, \quad x = -4 + 3y$$

$6x + y - 2 = 0$, $x - 3y + 4 = 0$
are the two straight lines.

② #4 $10x^2 - 23xy - 5y^2 - 29x + 32y + 21 = 0$ ——— ①

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

Here $a = 10, h = -\frac{23}{2}, b = -5, g = -\frac{29}{2}, f = 16, c = 21$

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = \begin{vmatrix} 10 & -\frac{23}{2} & -\frac{29}{2} \\ -\frac{23}{2} & -5 & 16 \\ -\frac{29}{2} & 16 & 21 \end{vmatrix}$$

$$= 10(-105 - 256) + \frac{23}{2}(-\frac{483}{2} + 232) - \frac{29}{2}(-184 + \frac{145}{2})$$

$$= -3610 + \frac{23}{2}(\frac{-483 + 464}{2}) - \frac{29}{2}(\frac{-368 + 145}{2})$$

$$= -3610 + \frac{23}{2}(-\frac{19}{2}) - \frac{29}{2}(\frac{-223}{2})(-\frac{513}{2})$$

$$= -3610 - \frac{437}{4} + \frac{14877}{4}$$

$$= \frac{-14440 - 437 + 14877}{4} = 0$$

The given equation represents the pair of lines.

$\Rightarrow$

$$10x^2 + x(-29 - 23y) + (-5y^2 + 32y + 21) = 0$$

$$10x^2 - x(23y + 29) + (-5y^2 + 32y + 21) = 0$$

$$x = \frac{(23y + 29) \pm \sqrt{(23y + 29)^2 - 40(-5y^2 + 32y + 21)}}{20}$$

$$= \frac{23y + 29 \pm \sqrt{529y^2 + 841 + 1334y + 200y^2 - 1280y - 840}}{20}$$

$$= \frac{23y + 29 \pm \sqrt{729y^2 + 54y + 1}}{20}$$

$$\begin{aligned}
 &= \frac{23y+29 \pm (27y+1)}{20} \\
 &= \frac{23y+29+27y+1}{20}, \frac{23y+29-27y-1}{20} \\
 &= \frac{50y+30}{20}, \frac{-4y+28}{20} \\
 &= \frac{5y+3}{2}, \frac{-y+7}{5}
 \end{aligned}$$

Hence the lines are

$$2x - 5y - 3 = 0, \quad 5x + y - 7 = 0$$

③ #5.

$$6x^2 - 15y^2 - xy + 16x + 24y = 0 \quad \text{--- ①}$$

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + C = 0$$

Here $a = 6, h = -\frac{1}{2}, b = -15, g = 8, f = 12, C = 0$

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & C \end{vmatrix} = \begin{vmatrix} 6 & -\frac{1}{2} & 8 \\ -\frac{1}{2} & -15 & 12 \\ 8 & 12 & 0 \end{vmatrix}$$

$$\begin{aligned}
 &= 6(-144) + \frac{1}{2}(-96) + 8(-6 + 120) \\
 &= -864 - 48 + 912 \\
 &= 0
 \end{aligned}$$

i.e ① Represents the two lines.

$$\Rightarrow 6x^2 + x(-y+16) + (-15y^2+24y) = 0$$

$$x = \frac{-(-y+16) \pm \sqrt{(-y+16)^2 - 4(6)(-15y^2+24y)}}{2(6)}$$

$$x = \frac{(y-16) \pm \sqrt{y^2 + 256 - 32y + 360y^2 - 576y}}{12}$$

$$x = \frac{y-16 \pm \sqrt{361y^2 - 608y + 256}}{12}$$

$$x = \frac{y-16 \pm (19y-16)}{12}$$

$$\Rightarrow x = \frac{y-16+19y-16}{12}, \quad x = \frac{y-16-19y+16}{12}$$

$$12x - 20y + 8 = 0, \quad 12x + 18y = 0$$

$$3x - 5y + 8 = 0, \quad 2x + 3y = 0$$

Are the two straight lines.

For what value of λ will each of the following equations represent a pair of straight lines.

① # 6.

$$\lambda x^2 - 10xy + 12y^2 + 5x - 16y - 3 = 0$$

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + C = 0$$

Here $a = \lambda, h = -5, b = 12, g = \frac{5}{2}, f = -8, C = -3$

The given equation represents two straight lines if,

$$\begin{vmatrix} \lambda & -5 & \frac{5}{2} \\ -5 & 12 & -8 \\ \frac{5}{2} & -8 & -3 \end{vmatrix} = 0$$

$$\Rightarrow \lambda(-36 - 64) + 5(15 + 20) + \frac{5}{2}(40 - 30) = 0$$

$$-100\lambda + 175 + 25 = 0$$

$$-100\lambda = -200$$

$$-\lambda = -2$$

$$\lambda = 2$$

Hence to represent a pair of straight lines λ must be equal to 2.

② # 7

$$\lambda xy + 5x + 3y + 2 = 0 \quad \text{----- (1)}$$

Here $a = 0, h = \frac{\lambda}{2}, b = 0, g = \frac{5}{2}, f = \frac{3}{2}, C = 2$

The given equation represents two straight lines if

$$\begin{vmatrix} 0 & \lambda/2 & 5/2 \\ \lambda/2 & 0 & 3/2 \\ 5/2 & 3/2 & 2 \end{vmatrix} = 0$$

$$-\lambda/2(\lambda - \frac{15}{4}) + 5/2(\frac{3\lambda}{4}) = 0$$

$$-\lambda^2/2 + 15\lambda/8 + 15\lambda/8 = 0$$

$$-4\lambda^2 + 15\lambda + 15\lambda = 0$$

$$-4\lambda^2 + 30\lambda = 0$$

$$-2\lambda^2 + 15\lambda = 0$$

$$\lambda(-2\lambda + 15) = 0 \Rightarrow \lambda = 0 \text{ or } \lambda = +15/2$$

If $\lambda=0$, the given equation is linear and represent a straight line. Hence a point of lines $\lambda = \frac{15}{2}$ for (1) represent.

Q #8

$$4x^2 - 9y^2 - 2(8+\lambda)x - 18y = 29 + 2\lambda \quad \text{--- (1)}$$

$$4x^2 - 9y^2 - 16x - 2\lambda x - 18y - 29 - 2\lambda = 0$$

$$4x^2 - 9y^2 - 2x(8+\lambda) - 18y - 29 - 2\lambda = 0$$

Here $a = 4, h = 0, b = -9, g = -(8+\lambda), f = -9, c = -(29+2\lambda)$

$\therefore$ (1) Represents two straight lines if

$$\begin{vmatrix} 4 & 0 & -8-\lambda \\ 0 & -9 & -9 \\ -8-\lambda & -9 & -29-2\lambda \end{vmatrix} = 0$$

$$4(-9(-29-2\lambda) - 81) + (-8-\lambda)(-(-9(-8-\lambda))) = 0$$

$$4(261 + 18\lambda - 81) + (-8-\lambda)(-72 - 9\lambda) = 0$$

$$1044 + 72\lambda - 324 + 576 + 72\lambda + 72\lambda + 9\lambda^2 = 0$$

$$9\lambda^2 + 216\lambda + 1296 = 0$$

$$\lambda^2 + 24\lambda + 144 = 0$$

$$(\lambda + 12)^2 = 0$$

$$\lambda + 12 = 0$$

$$\lambda = -12$$

Hence $\lambda = -12$

Find the angle b/w each of the following pairs of lines.

Q #9

$$x^2 - 2xy \tan \theta - y^2 = 0 \quad \text{--- (1)}$$

$$ax^2 + 2hxy + by^2 = 0$$

Here $a = 1, h = -\tan \theta, b = -1$

$$\therefore a = 1, b = -1$$

$$\therefore a + b = 0$$

Lines generated by (1) are $\perp$ i.e the angle b/w the lines = 90°

Q #10.

$$3x^2 + 7xy + 2y^2 = 0 \quad \text{--- (1)}$$

$$ax^2 + 2hxy + by^2 = 0$$

Here

$$a = 3, \quad h = 7/2, \quad b = 2$$

$$\tan \theta = \frac{2\sqrt{h^2 - ab}}{a+b}$$

$$= \frac{2\sqrt{49/4 - 6}}{5}$$

$$= \frac{2\sqrt{\frac{49-24}{4}}}{5} = \frac{2\sqrt{25/4}}{5} = \frac{2 \cdot 5/2}{5} = 1$$

$$\theta = \tan^{-1}(1)$$

$$\theta = \pi/4$$

i.e. the angle b/w the lines is $\frac{\pi}{4}$.

Q #11

$$11x^2 + 16xy - y^2 = 0 \quad \text{--- (1)}$$

$$a = 11, \quad h = 8, \quad b = -1$$

$$\tan \theta = \frac{2\sqrt{64+11}}{11-1} = \frac{2\sqrt{75}}{10} = \frac{2 \times 5\sqrt{3}}{10}$$

$$\tan \theta = \sqrt{3}$$

$$\theta = \tan^{-1}(\sqrt{3})$$

$$\theta = 60^\circ$$

i.e. the angle b/w the curves is 60° .

Q #12

$$x^2 + 4xy + y^2 - 6x - 3 = 0$$

$$a = 1, \quad h = 2, \quad b = 1$$

$$\tan \theta = \frac{2\sqrt{4-1}}{2} = \frac{2\sqrt{3}}{2} = \sqrt{3}$$

$$\tan \theta = \sqrt{3}$$

$$\theta = 60^\circ$$

i.e. the angle b/w the lines = 60°

Q #13

$$6x^2 + xy - y^2 - 21x - 8y + 9 = 0$$

Here $a = 6$, $b = -1$, $h = \frac{1}{2}$

$$\begin{aligned} \tan \theta &= \frac{2\sqrt{\frac{1}{4} + 6}}{6-1} = \frac{2\sqrt{\frac{4+24}{4}}}{5} \\ &= \frac{\sqrt{28}}{5} = \frac{5}{5} = 1 \end{aligned}$$

$$\theta = \tan^{-1}(1)$$

$$\theta = 45^\circ$$

i.e. the angle b/w the lines is 45° .

Q #14 Show that $16xy - 6x + 8y - 3 = 0$ represents a pair of straight lines. Also prove that this together with the coordinate axes form a rectangle and find the area enclosed by the rectangle.

Soln.

$$16xy - 6x + 8y - 3 = 0 \quad \text{--- (1)}$$

$$2x(8y - 3) + (8y - 3) = 0$$

$$(2x+1)(8y-3) = 0$$

$$\Rightarrow 2x+1=0, \quad 8y-3=0$$

$\therefore$ (1) has been factorized into two ^{linear} factors

$\therefore$ The resulting equations represent the two straight lines.

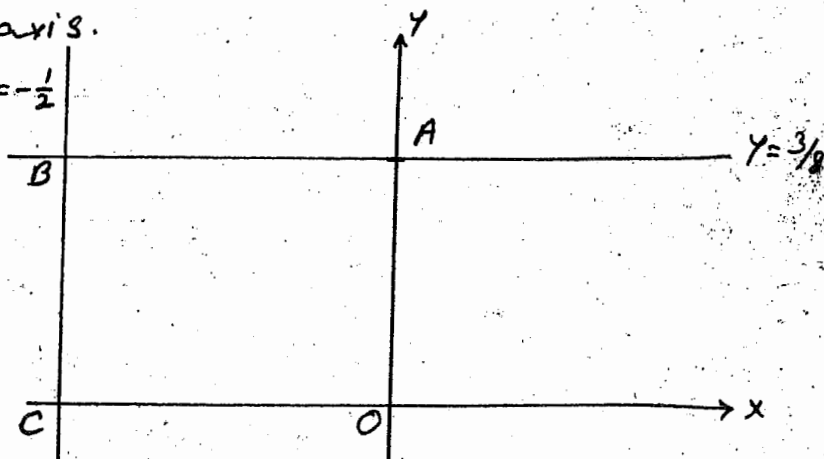
We can write $2x+1=0$ and $8y-3=0$ in the form $x = -\frac{1}{2}$, slope $\parallel$ to y-axis and $y = \frac{3}{8}$ $\parallel$ line $\parallel$ to x-axis.

Area of the Triangle

$$= 10 C |OA|$$

$$= |-\frac{1}{2}| | \frac{3}{8} |$$

$$= \frac{3}{16} \text{ Sq. Unit}$$



☺ #15. Show that an equation of rectangular hyperbola $x^2 - y^2 = 1$ referred to its asymptotes as axis is $x'y' = -\frac{1}{2}$

Soln

The given rectangular hyperbola is

$$x^2 - y^2 = 1 \quad \text{--- (1)}$$

We know that the equations of the transformations are

$$x = x' \cos \theta - y' \sin \theta \quad \text{--- (2)}$$

$$y = x' \sin \theta + y' \cos \theta \quad \text{--- (3)}$$

For the eq. referred to its asymptotes, we know that $\theta = \pi/4$

$$2) \Rightarrow x = x' \cos(\pi/4) - y' \sin(\pi/4)$$

$$x = \frac{x'}{\sqrt{2}} - \frac{y'}{\sqrt{2}}$$

$$x = \frac{1}{\sqrt{2}} (x' - y') \quad \text{--- (4)}$$

$$3) \Rightarrow y = x' \sin(\pi/4) + y' \cos(\pi/4)$$

$$y = x' \frac{1}{\sqrt{2}} + y' \frac{1}{\sqrt{2}}$$

$$y = \frac{1}{\sqrt{2}} (x' + y') \quad \text{--- (5)}$$

Putting these values in (1), we have

$$\frac{1}{2} (x' - y')^2 - \frac{1}{2} (x' + y')^2 = 1$$

$$x'^2 + y'^2 - 2x'y' - x'^2 - y'^2 - 2x'y' = 2$$

$$-4x'y' = 2$$

$$x'y' = -\frac{1}{2}$$

Angle b/w the asymptotes and the axes is $\pi/4$

Note The asymptotes $y = \pm x$ or regarded as axes. Keeping these axes in view the equation $x^2 - y^2 = 1$ is reduced to

$$x'y' = -\frac{1}{2} \text{ by proceeding as above.}$$

Analyze and graph the conic represented by each of the following equations.

☺ #16 - ☺ #25

For ☺.16 - ☺.25

See the graph Book.